



LUDWIG WITTGENSTEIN

NOTES ON LOGIC

Introduction, arrangement,
and commented notes by
João José R. L. de Almeida

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Notes on Logic

by

Ludwig Wittgenstein

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Abbreviations of Wittgenstein's works*

AC	Anotações Sobre as Cores (2009b)
AWL	Wittgenstein's Lectures. Cambridge, 1932-1935 (1979)
BB	The Blue and Brown Books (1969a)
BLF	Briefe an Ludwig Von Ficker (1969d)
BM I	Bemerkungen I – TS 228 (2000)
BM II	Bemerkungen II – TS 230 (2000)
BT	The Big Typescript (2005)
CLD	Wittgenstein in Cambridge. Letters and Documents (2008)
CV	Culture and Value (1998)
D	Ditados do Nachlass (2000)
DB	Denkbewegungen. Tagebücher 1930-1932, 1936-1937 (1997)
LC	Lectures and Conversations on Aesthetics, Psychology and Religious Belief (1966)
LDP	Ludwig Wittgenstein: Dictating Philosophy (2020)
LE	A Lecture on Ethics (1993a)
LFM	Wittgenstein's Lectures on the Foundations of Mathematics (1976)
LO	Letters To C. K. Ogden (1973)
LPP	Wittgenstein's Lectures on Philosophical Psychology (1988)
LWL	Wittgenstein's Lectures, 1930-1932 (1980a)
LWPP I	Last Writings on the Philosophy of Psychology I (1982)
LWPP II	Last Writings on the Philosophy of Psychology II (1992)
M	Wittgenstein's Lectures in 1930-1933 (Moore, 2004a)
MS	Nachlass Manuscript (2000)
NB	Notebooks 1914-1916 (1969a)
NL	Notas Sobre Lógica
NLPD	Notes For Lectures on Private Experience and Sense Data (1993b)
OC	On Certainty (1969c)
ORO	Observações Sobre O Ramo de Ouro de Frazer (2022c)
PG	Philosophical Grammar (1974)
PI	Philosophical Investigations (2022a)
PPF	Philosophy of Psychology: A Fragment (2009a)
PPO	Public and Private Occasions (2003b)
PR	Philosophical Remarks (1975)
RFM	Remarks on Foundations of Mathematics (2022b)
RPP I	Remarks on The Philosophy Of Psychology I (1980b)
RPP II	Remarks on The Philosophy Of Psychology II (1980c)
TLP	Tractatus Logico-Philosophicus (2001a)
TS	Nachlass' Typescript (2000)
VW	The Voices of Wittgenstein. The Vienna Circle (2003a)
WLC	Wittgenstein Lectures, Cambridge 1930-1933 (2016)
WWC	Wittgenstein's Whewell's Court Lectures. Cambridge, 1938-1941 (2017)
WVC	Wittgenstein e o Círculo de Viena (2022d)
Z	Zettel (1967a)

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* Abbreviations for Wittgenstein's works are based on the initial letters of the titles of published books, as listed in the bibliography. Texts not published in book form that are part of the literary estate (*Nachlass*) are abbreviated using the names by which they are traditionally known: BM for *Bemerkungen*; MS for Manuscripts; TS for Typescripts; and D for Dictates. The literary estate was published by the Wittgenstein Archives at the University of Bergen in a collection known as the Bergen Electronic Edition (BEE). It is currently on-line as the Wittgenstein Source, at the University of Bergen: <http://www.wittgensteinsource.org/>

PRESENTATION

The “Notes on Logic” (NL) present one of the most curious cases in the editorial fortune of excerpts from Wittgenstein’s literary legacy - the so-called Nachlass. What immediately draws attention when situating the NL within his philosophical estate is a characteristic form of editorial intervention, perhaps stemming from an unfamiliarity with the writer’s style and attitude. In this sense, what causes surprise regarding the NL is precisely the homogeneity of their fate: the NL undergo the same kind of normalization as all other published texts by Wittgenstein - except for the TLP, of course. Originally, the NL were a typescript of three manuscripts (MSS 1, 3, and 4), supplemented by the outcome of a discussion devoted to another (MS 2). However, Bertrand Russell diligently transformed the work into an almost continuous, topicalized text. In this new format, it conveys the distinct impression of a finished task corresponding to a well-defined stage of study and discussion - something akin to a section of a prospective book project.¹

I shall refer to this assumption of completion - which may or may not exist among readers of the philosopher - as “Wittgenstein’s books.” The existing books by Wittgenstein, such as *Philosophical Remarks*, *Philosophical Grammar*, *Remarks on the Foundations of Mathematics*, *Remarks on the Philosophy of Psychology*, etc., are generally collections of passages drawn from distinct manuscripts and typescripts, brought together in these volumes based on thematic similarity and the continuity of ideas. Indeed, the author himself suggested that his literary executors publish his legacy as they saw fit (cf. “The PI as an Unfinished Work,” in: Wittgenstein (2025), pp. 13–57). Yet, published as uninterrupted textual blocks and presented as concluded relative to their respective preliminary stages of research, they may, as a side effect, reinforce the presumption of being “Wittgenstein’s books.”

Rather than assuming closure and resolution as the conventional rule for presenting a philosophical text, I prefer to see in the NL the same design found in virtually all of the author’s work: a form of composition that generates difficulties in comprehension, initially brought about by an unsystematic clustering of condensed, lapidary fragments that consistently characterized his literary style. Precisely because they are not “books” in the commonly presumed sense, but open, gappy, fragmentary composites, they can trigger a clash of expectations. We are confronted with a writing style that goes hand in hand with an almost irremediable proliferation of misunderstandings concerning the author’s precise thought across passages that explore concepts and methods in multiple, disordered directions.

Whether these stylistic effects were accidental or intentional remains open to debate,

and arguments can be advanced for both sides; personally, I lean toward the latter.² Several reasons support this view. For instance, in MS 109, written in 1930, our author explicitly reveals his intent to compose a book intelligible only to those akin to his spirit. Wittgenstein refers to what he characterizes as a spirit of “culture,” which, in this case, confronts a spirit of “civilization.” “Culture” would be an aggregation of forces organized into a totality capable of assigning a defined place to each of its members. For individuals sheltered beneath this cultural mantle, energies are not dissipated or worn down by alien illusions of expansion and development typical of imported notions such as “technology and progress.” For Wittgenstein, the idea of civilization offers no communal cohesion, and its most characteristic symptom is the disappearance of art. He thus wishes to exclude from his “book” all readers who do not share his spirit of resistance against the idea of civilization and progress. It is not that these individuals would be unable to read the text; on the contrary, it is entirely readable. However, they would remain permanently excluded from grasping its deeper content: “The book must automatically produce the separation between those who understand it and those who do not” (MS 109, p. 208). Clearly, this “book” is not—nor could it ever be—a book in the traditional philosophical sense: a completed task corresponding to a well-defined stage of study and debate.

By 1931, Wittgenstein explicitly acknowledged the writing style he had been practicing up to that point, openly embracing it as a philosophical method:

The uneasiness in philosophy comes from the fact that philosophers see it incorrectly - or see it the wrong way -, that is, as if it were divided into longitudinal (infinite) strips, and not into horizontal (finite) strips. This adjustment of conception gives rise to the *greatest* difficulty. They want, so to speak, to encompass the infinite strip and complain that this is not possible little by little. Certainly not, if by “little” one understands an endless longitudinal strip. But it is possible if one views a transverse strip as a fragment, a complete and defined fragment. - But then we would never finish our task! Certainly not, for it has no end. (MS 112, pp. 71v-72r)³

What this excerpt suggests is a gradual, piecemeal, and more or less erratic progression - progressive, yet non-evolutionary. It is not a rectilinear, compounding movement. It is as though the proper conditions for deciding which new direction to take emerge only at the end of each stretch, highlighting the typical motion of someone discerning the contours of a landscape while traversing it. Only after several winding, complex incursions - punctuated by occasional stops and episodic returns to the starting point - do sufficient convictions form to continue the journey. Thus, what seems to drive this wandering approach of the lost traveler are certain conditions of authenticity and ethical commitments, graspable only through an overview of the distinct paths traversed along the way - that is, through an a posteriori assimilation of the routes traced during a disoriented course. This suggests a *praxiological method* grounded in contextual vicissitudes, reflecting a practice guided solely by the intention to arrive, but still without precise mapping at hand. Therefore, unless we view this philosophical practice through the lens of cross-strips, we will fail to understand it fully, perhaps even becoming prisoners of philosophical uneasiness.

Around 1940, Wittgenstein also recognized the precise manner in which he expected his text to affect its readers:

1. Wittgenstein’s “books” - such as *Philosophical Remarks*, *Philosophical Grammar*, *Remarks on the Foundations of Mathematics*, *Remarks on the Philosophy of Psychology*, etc. - are, in general, collections of excerpts drawn from various manuscripts and typescripts, assembled into these volumes based on the similarity and continuity of the ideas. The author himself suggested that his literary heirs publish his legacy as they saw fit: cf. “The Philosophical Investigations as an Unfinished Work,” in: Wittgenstein (2025), pp. 13–57.

2. The reader can find good discussions regarding Wittgenstein’s writing style and philosophical method in Pichler (2007 and 2023), as well as in Kanterian (2012). However, in MS 109, p. 208, our author explicitly reveals that he wishes to compose a book that can be understood only by those who are kindred spirits: “The book must automatically bring about a separation between those who understand it and those who do not.”

3. This passage, composed on November 1st, 1931, was incorporated into BT (TS 213, pp. 431–432), and later also became part of the first handwritten version of the PI (MS 142, pp. 120–121).

I think my originality is (if that is the right word) an originality belonging to the soil, not the seed (perhaps I have no seed of my own). Sow a seed in my soil and it will grow differently than it would in any other soil. Freud's originality was also, I believe, of this kind. I have always believed—without knowing why—that the real seed of psychoanalysis came from Breuer, not Freud. Breuer's seed, of course, can only have been tiny. (Courage is always original.) (MS 162b, pp. 59v–60v)

Evidently, he expects his text to exert a unique therapeutic effect on each reader, closely resembling the outcome of a completed psychoanalysis. He makes it clear, even in late remarks such as this one from 1948, that he writes exclusively for a “noble sort of reader”:

[For the preface] It is not without reluctance that I hand the book over to the public. The hands into which it falls will, for the most part, not be those I like to imagine. May it soon—this is my wish—be completely forgotten by the philosophical journalists, & thus remain preserved for a nobler sort of reader. (MS 136, p. 81a).

In my translations, I typically embrace this stylistic/methodological preference as part of the rationale for proposing a “physiognomy of Wittgenstein's texts.”⁴ I maintain that the reading effects of this stylistic practice should be considered more seriously in the arrangement and translation of excerpts from the *Nachlass*. For reasons relating to practical adaptation during composition, the literary strategy evident in the arrangement of sentences and paragraphs, and overall aesthetic structure - matters addressed in the work cited in the footnote - I believe these outcomes are an integral part of the author's philosophical project itself.

What is particularly unusual about the text of the NL, however, is that it already reveals a mode of operation roughly corresponding to all typescript transcriptions of his notebooks throughout his extensive 38-year philosophical career (1913–1951). This entire aesthetic grammar, in which eloquence carries a distinctly practical rather than explicitly stated emphasis, culminates in what would later become a self-reflection on his method: “In art there is nothing as good as: saying nothing” (MS 156, p. 57r) - a conclusion reached after five continuous pages of notes on music, sculpture, the natural history of humanity, literature, and psychoanalysis.

In light of this, I would like to highlight the following: one of the most evident consequences of reading extremely restrained and unfinished texts, such as this one from the NL, consists basically in the fulfillment of two fates. The first occurs when they are published, as I said, in book form. The cognitive dissonance between the book that is expected and the album that actually takes shape tends to trigger objections incongruous with the reality of the facts. Once bound, these fragmentary texts furtively convey the idea of a finished, concluded work, complete in itself. As if the author's thought, in its very germ, did not demonstrate exactly the opposite: a merely initiated course of nomadic wandering through a labyrinth of intersecting paths (cf. PI § 203). Unlike the presumption of evolutionary improvement, perhaps occasioned by the act of typing the manuscripts; unlike the impregnation in our distracted mind of the image of a rigidly continuous and conclusive straight line, which generally describes the completion of a typically Western philosophical work, Wittgenstein's texts show, from the very first moment, from the first time one of his manuscripts was transformed into typescript, a digressive movement and a segmentary character,

4. See Almeida (2015) for an approach to the concept of “physiognomy of the text.”

manifestly different from the habitual tendency, and duly acknowledged in the currently published preface to the PI as “a journey through a vast domain of thoughts, here and there, in all directions.”

As for the second fate, it adds to the first an obstacle to the immediate understanding of what the author intends to say; yet, more than that, it introduces an even greater difficulty concerning what the author intends to show by what he writes. These are not merely divergent interpretations, common in philosophers such as Kant, Hegel, or Nietzsche, but a kind of deliberate disarrangement of perspectives, if one may put it that way, spreading simultaneously in every direction with the same speed as fire running through dry brush. A more or less chaotic situation, when compared with the interpretive divergences of other Western philosophies, even the most fragmentary ones, such as those of the already mentioned Nietzsche, but also those of Kierkegaard or Benjamin. Independently, however, of the specific reasons for this strange phenomenon, already discussed in celebrated texts such as those by Biletzki (2003) and Stern (2006), in this episode of the NL the case appears from the outset through the “organizing” intervention of Bertrand Russell (I mean: in the sense in which Russell presupposed what the organization of philosophical material would be) upon the text effectively expressed in notebooks. This went so far as to transform it, at the beginning of 1914, into a writing subdivided into topics and rationally systematized from the simplest to the most complex. Russell was, even in this sense, a paradigmatic predecessor of the author's literary heirs: Elizabeth Anscombe, Rush Rhees, and Georg von Wright. They too structured and arranged, according to their legitimate assumptions, the publication of Wittgenstein's legacy by creating titles of books topicalized by themes which, even unintentionally, suggest an idea of the cumulative continuity of concepts and their internal concatenations. The result may have been to transform the “literary composite” (Paul, 2007, p. 23) into a kind of rigid and indifferent rail, extended linearly toward an uninterrupted journey to infinity (cf. PI § 218). What had always had the musical form typical of a *ritornello*, of the intermittent tune of a refrain interwoven with harmonic architecture without any apparent purpose, bringing back to the listener an already familiar landscape, causes that, once the excerpt is published as a book, what shows itself without anything needing to be said becomes obscured. The multiform rootedness spread in all directions, the preferentially cloudy landscape, thereby loses some of its literary prominence, whether through the burial imposed by the organizing layer of the literary heirs and editors of the texts, or through the overpowering effect instilled by the more immediate perspective, in our historical period, of a reasoning affected by the idea of progress.

The physiognomy of the Wittgensteinian text does not show a uniform and rectilinear movement; it presents a state of deposition or sedimentation in the manner of a “precipitate of philosophical investigations,” as consecrated by the preface to the PI. This is how, I would like to presume, Wittgenstein's writings appear if we contemplate them from the genealogical perspective of the *Nachlass*.⁵

From this point of view, how, then, did the NL arise? They were initially conceived in a letter addressed to Russell on September 20, 1913, in which Wittgenstein asked for a meeting in order to present all the elaborations he had carried out up to that point and, if possible, to take new notes in Russell's presence (cf. McGuinness, 2008, p. 46, letter 22). Until then, the young philosopher had habitually discussed the results of his research either in

5. The *Nachlass* is currently published by the Wittgenstein Archives at the University of Bergen, with free access at: <http://www.wittgensteinsource.org>.

personal conversation or through letters sent from Austria, where he often traveled because of his father's convalescence or for festivities such as Christmas and Easter. He also sent letters to Russell from Norway, from which that last request came, after he first traveled there in the summer of 1913 on a restful trip with his friend David Pinsent.

Wittgenstein also maintained this habit of discussing his latest results with Frege. The young philosopher made three visits to Jena before the war, in 1911, 1912, and 1913 - a fact of which Russell was completely unaware. The last of these, in December 1913, was also to discuss the results of the NL (cf. McGuinness, 1988, pp. 190–191).

But the urgency of this request to Russell came from an anxious feeling that haunted him at the time: the constant fear that he might die suddenly and all his work be definitively lost.⁶ It also allowed the idea to arise in him that he should isolate himself, by returning to Norway, in order to perfect and finish this work without any kind of distraction or disturbance arising from environmental and social contingencies that troubled him. The notes to be taken in October were therefore intended to be a legacy. If not the definitive legacy of his philosophical work, at least one with the degree of consistency he had been able to reach up to that period. This did not prevent the conception that he should devote himself much more to these first elaborations if he survived.

We must acknowledge that this degree of consistency gives the effort an autonomy of its own, independent of any later development, and for that reason it deserves to be read on its own terms. Michael Potter produced a remarkable study and commentary on the NL precisely because he was convinced of this (cf. 2009, pp. 1 and 261).

Wittgenstein came to Cambridge to meet Russell on October 2, 1913, an occasion on which he managed to present parts of his manuscript notes to him. Russell thought the ideas were good, but very difficult to understand without a great effort of mental concentration (cf. Potter, 2009, p. 263). Perhaps for this reason Wittgenstein promised Russell that he would leave in writing everything he had done up to that point before departing again, this time alone, for Norway. In fact, during a visit to Pinsent's parents in the city of Birmingham, Wittgenstein found, at the Berlitz language school, on October 7, 1913, a typist to whom he could dictate extracts from his diaries in German (Potter, 2009, p. 264). The next day, October 8, 1913, Wittgenstein was already back in Cambridge to continue discussing his new ideas with Russell. As those discussions became increasingly heated and therefore much more difficult to follow with full comprehension, both agreed to meet once more the following day. Russell arranged for Philip B. Jourdain's secretary, Miss Harwood,⁷ to take notes of these discussions. Jourdain was Russell's friend, and also a logician and mathematician working at Cambridge. The German typescript, composed in Birmingham, would come into Russell's hands sometime after these events. Thus Russell must have had two typescripts available: the Birmingham one, which he translated by hand into English, and the Cambridge one, typed by Jourdain's secretary and supplemented with some handwritten notes of what had been produced in the exhausting discussions between the two thinkers strictly over those two days—but also through subsequent letters (Potter, 2009, p. 265).

What material is available today in the *Nachlass*? We currently have two sets of papers. The first, designated TS 201a1, contains 7 typed pages with extensive handwritten annotations in the body of the text (the handwriting is Wittgenstein's). The annotations on the typescript are intended to correct words and reformulate some previously typed ideas.

6. Wittgenstein's peculiar state of mind at that time is well described in Chapter 6 of McGuinness (1988, pp. 179–203).

7. Her name is mentioned in a letter from Wittgenstein to Russell (2008, p. 52, n. 27).

These typed pages are followed by another 24 pages entirely handwritten, these in Russell's hand, who probably copied and translated the diaries already typed in Birmingham.⁸

The second set of papers, designated TS 201a2, contains no more handwritten pages. All the content that appeared in the first set as a mixture of typescript and manuscript now appears fully rendered by typewriter, with no modifications in relation to the previous text.

However, looking more closely at the two sets of papers (TS 201a1 and TS 201a2), we see that the second does indeed contain handwritten annotations in the margins of the text that were not present in the first set. These are Roman numerals placed on the left side of the sheets, alongside almost every paragraph of the text, in addition to the word “preliminary” inserted on three occasions, on pages 22, 23, and 24. These Roman numerals and the word “preliminary” correspond, in fact, to a topicalization of the content whose list appears on a handwritten page at the end of TS 201a1, in Russell's hand.

The best available interpretation of this configuration seems to me to be Potter's (2009, pp. 265–271). I do not know whether there are in fact any others, apart from McGuinness's first attempt in 2002 (cf. pp. 243–258). Potter assumes that MS 2 and the Summary, because they have very similar writing styles, must be transcriptions of what Wittgenstein explained to Russell in person between October 8 and 9, 1913. Much of MS 2 also seems to reformulate material contained in MSS 1, 3, and 4.

MSS 1, 3, and 4, in turn, would be English translations of the typescript produced in Birmingham, now lost. Potter argues that the type of corrections that appear abundantly in this trio of manuscripts, such as crossed-out words replaced by others - a kind of rectification completely absent from MS 2 - would be strange occurrences if they were merely copies of what was being dictated. These corrections look much more like rectifications of translated text, not dictated text (cf. Potter, 2009, p. 266). Yet another significant conjecture is that there is an entire passage of the *Prototractatus* (MS 104, pp. 28–34) that seems to have been taken directly from MSS 1, 3, and 4. This confirmatorily attests to the fact that Wittgenstein took MSS 1, 3, and 4 with him to Austria. On the other hand, the *Prototractatus* never comes close to the content of the Summary and MS 2.

If this hypothesis is true, then why would Russell have inserted MS 2 between MS 1 and MSS 3 and 4, that is, in the middle of the real sequence of the Birmingham typescript? Potter's speculation is that Wittgenstein brought only MS 1 with him to the first meeting with Russell. It contains only 600 words, which would correspond perfectly to a work session of a few hours on the first and second days of the face-to-face meeting with Russell. The rest of the typed package, much more extensive, would have been received days later by Wittgenstein's tutor, sent directly by the Birmingham typist, when Wittgenstein had already departed for Norway. Thus Russell would simply have repeated, in his translation and copy, the order in which he received them.

To complicate these successive retractions even further, more than 40 years after these events a publication of the “Notes on Logic” appeared, prepared by Harry Costello (cf. 1957), with configurations different from those we have seen so far. Costello was the teaching assistant appointed by Harvard to help Russell in his 1914 course at that university, a course for which Russell prepared with the understanding he had gained from the critical discussions he had had with Wittgenstein in those final months. In Costello's version of the NL, the Roman numerals and the annotation “preliminary,” which had previously been only marginal annotations in TS 201a2, became actual subtopics of the text, which now

8. This conjecture was proposed by both McGuinness (2002, pp. 245–246) and Potter (2009, pp. 265–266). I invite you, dear reader, to examine the reasoning presented there in support of this hypothesis.

acquired the order that had earlier only been suggested. It should also be noted that two crossed-out paragraphs in the present version (on pp. B19 and B20) do not appear in Costello's version. In the introduction to this publication, the assistant states that when Russell arrived at Harvard in 1914, he brought some notes and handwritten excerpts by Wittgenstein, and that he copied these texts dated "September, 1913" (Costello, p. 230).

According to the final paragraphs of McGuinness, 2002 (p. 258), Costello's version of the NL was a work of his own, carried out "on the basis of Russell's indications of the sections to which each remark should be assigned, as well as the titles of those sections" (p. 258). This conjecture fits with that of Biggs (1996, p. 10), who holds that Costello copied a rearrangement made by Russell of what was still in manuscript form. Nevertheless, Potter emphasizes that the copy was made from a typescript, not from a manuscript (2009, p. 273); he simply cannot determine whether Costello's copy resulted in another manuscript or another typescript (p. 272, note 28).

Well then, what would the physiognomy of the NL be from my perspective? It is that not even TS 201a1, the oldest available version, would count as an "original text." Nevertheless, in light of this design so common in Wittgenstein's writings, I think we should question the point of seeking "the most original" among them. Why do so? Would this original occupy the same place as a "fact" for someone seeking a neutral instance capable of verifying, with greater precision and rigor, the support for a hypothesis? In other words, would it function as an immaculate fixed point, embedded in an untouched space, by means of which we could measure the correctness of every representation made of it? I do not know whether this diligent search truly aims to arrive at the "original fact," nor would I even know how to discuss what constitutes a "philosophy of the search for the original"; but one thing I might perhaps take as certain: this kind of "original" is probably not available in the physiognomy of Wittgenstein's texts. What the writings of the Viennese philosopher show are continuous cycles, resumptions of processes at a new level of a spiral movement, like dawn, the duration of the day, dusk, night, and a new sunrise. These are contingencies proper to the temporal eventuality that so deeply affects what we might perhaps call "human experience." From a configuration rooted in bundles projecting in all dimensions, we have nothing more than the testimony of our "aspect-seeing." There is simply no place, no available room, for a unilateral view of such a complex literary structure.

It is in this sense that I have chosen TS 201a2 as the source text for the translation. This typescript is not only an unfinished work, typical of Wittgenstein, but it is also the last of the NL that contains, in the typed material, the corrections requested by the author and sent from Bergen. In addition, it carries Russell's organizing interventions, later faithfully incorporated by Costello's version. In this way, it also organically incorporates the structural misunderstanding that the TLP would finally bring to maturity by endowing the aesthetic primacy of showing over saying, transforming the totality of the work into a vision of the object *sub specie aeternitatis*. A form of vision possible only through the active complicity of the reader's will.

I therefore believe that this source text is the most representative of the idea of the physiognomy of the *Nachlass* that I defend here.

What shall we find in the content of the NL? Wittgenstein's principal achievement in this work was to present a "theory of symbolism" as a more independent metaphysical alternative, or, if we can translate the notion of independence into an equivalent sentence, as a concept more firmly anchored in human practice. It is within our common activities that operational metaphysics supposedly functions only as an auxiliary instrument of a prede-

finied purpose. In the NL, Wittgenstein completely dispenses with doctrinal metaphysics, the kind that claims to unveil a supposedly underlying and deeper structure of reality.

For someone who then postulated that philosophy consists only of logic and metaphysics, and that logic would in fact be its basis (TS 201a2, p. B22), this shift was important for two reasons: first, because Wittgenstein freed himself, in one stroke, from the naturalistic burden of Russell's logic and Frege's Platonic realism, both of which fostered the second kind of metaphysics; and, second, because he initiated the first attempt at what would become, during and shortly after the war years, a "picture theory."

At this moment, the word "theory" may be used here without hesitation, because that was precisely how he recognized his activity (cf. TS 201a2, p. A5). The praxiological distinction between "theory," or "doctrine" (*Lehre*), and "activity" (*Tätigkeit*), would only truly become consolidated in 1918, in the *Prototractatus*: "Philosophy is not a doctrine, but an activity," Wittgenstein would say (MS 104, p. 19); logic, likewise, would not be a doctrine, but "a mirror image of the world" (MS 104, p. 66). Both conceptions surface in the TLP, respectively in sections §§ 4.112 and 6.13.

Nevertheless, I would like to call attention to the fact that this "theory," although presented under that name, was already, in fact, much more than a conjecture capable of verification concerning a given order of phenomena, as we might generally describe the activity that characterizes scientific production. In the NL, instead, we witness a description of a common mode of human action; indeed, already a natural fact of human history, as would later be acknowledged (RFM PI§63, PI§142, PII§40, PIV§§11,13, PVI§49, PVII§17; PI §§ 25, 415). It was not the formulation of an empiricist attitude, typically representationalist, toward logic. In such a view, the mental world, charged with forming ideas from sense impressions, comes into contact with the empirical world, charged with presenting sensory stimuli for the formation of ideas. Nor was it, for logic, the empirical configuration of the rules of a real world, outside the empirical dimension itself and also outside the mental dimension, in which the foundation of mathematics verifies its real scientific objectivity. Such conceptions presuppose from the outset the existence of two or three sealed-off worlds—presuppositions that are very difficult to balance and sustain philosophically.

In other words, what I want to demonstrate here is that in the NL it is already possible to glimpse that language is, for Wittgenstein, a form of action internally interwoven with its representations. One may debate what the representationalist presuppositions of language would in turn be. One may question whether these would be precisely the most praxiological forms possible. But what this action is, what it is anchored in, what the nature is of the material that enables this acting, what its conditions of possibility are—these are no longer admissible questions in this work. As Potter clearly shows, for Wittgenstein, as early as 1912, logic has no content; it is entirely empty; there is nothing to be made known about it, nor is there any need for validation for its application (cf. Potter, 2009, pp. 61–62).

To understand this point, I believe it is necessary to distinguish two orders of problems. The first is to establish how Wittgenstein differs from Russell and Frege in this text. The second is to indicate precisely how, also in this text, he frames what he calls "language." Let us not forget that, in this philosophy, it is the linguistic—and therefore symbolism—that ensures that the final decision does not fall back into the psychological, or, to be clear, into subjectivity, so that we have at our disposal some sense capable of continuing the exchange of communicational values.

How does this work? In the NL, Wittgenstein will attest, albeit timidly, to what on December 25, 1914 (NB, p. 37) he can already assert with the phrase: "... there may be no

logical constants” (TS 201a2, p. B5).

For Frege and Russell, logic was almost a kind of “physics of thought.” I mean that it did, in fact, possess a “substantial content.” Not only was it amenable to theoretical treatment, but above all it was capable of forming an appearance of solidity to validate those theoretical operations. Logical constants (such as negation $\sim$, conjunction $\wedge$, disjunction $\vee$, implication $\rightarrow$, the biconditional $\equiv$, as well as the universal $\forall$ and existential $\exists$ quantifiers), expressions that preserve a fixed meaning across all interpretations, functioned as entities or functions that referred to *something real* in the domain of logic. Frege, in particular, viewed the constants as functions that mapped truth-values. Negation or the conditional were “logical objects” of a special kind. Russell, in turn, considered logical constants to be inalienable components of propositions. We understand a proposition such as “p and q” simply because within it we have a kind of “knowledge by acquaintance” with the logical object. In these perspectives, logic becomes a “science of logical objects,” even if extremely abstract ones.

Wittgenstein, however, sets out in this interval to extract from ordinary language only the logical structure incorporated within it, like bones that support the musculature and the body’s movements:

The structure of the proposition has to be recognized; the rest comes by itself. But ordinary language conceals the structure of the proposition: in it, relations look like predicates, predicates look like names, etc. (TS 201a2, p. B23).

This logical structure was what he regarded as “language.” Ordinary language, in turn, functions here as a map of logical necessities. When we speak, declare, or assert anything whatever about experience, that speech, declaration, or assertion - concrete sound movements and articulations - whether they are embedded not only, as one might expect, in assertions, but also in *judgments, commands, and questions*, are *immediately recognized*. Recognized in what way? By the *propositional form* underlying those movements. If natural language allows negation (“not-p”), this is a sign that there is a logical operation internal to the symbolic act taking place there. The parameter here is not the form of the ordinary sentence, but the function it performs without noticing it. The question “what is being negated?” is what ensures that logic is not merely a game of symbol manipulation (as in purely formal systems), but something anchored in sense. Logic is not something we superimpose upon the world, but something already operating latently in ordinary language.

Perhaps it is not wrong to say that in the TLP this confidence in ordinary language became much more technical. In section § 4.002, he says that “Everyday language is a part of the human organism and is no less complicated than it,” and that “It is humanly impossible to gather immediately from it the logic of language.” But for the moment, in the NL, there is still a sharing, between ordinary language and logic, of the same deep structure embedded in speech acts and in beliefs or judgments: “Judgment, command, and question are all on the same level; but they all have in common the propositional form, which is what interests us” (TS 201a2, p. B23).

The presence of “commands and questions” in this sentence does not show that Wittgenstein had slipped from the restricted sphere of the assertive/descriptive function of language - the only one that can be “true or false” - into the sphere of something with a praxiological accent, such as intentional movements, actions of the will, or human techniques embodied in customs. Whether propositional form was an innate human capacity,

or whether it was something normative, like a technique socially consolidated by practice, was a matter of no interest to logic—and it did not interest him either. But why? Probably because he never conceived philosophy as a cognitive task, but perhaps as a charge of eminently ethical character. Let us note, however, that this same anti-cognitivist philosophical attitude would later lead him to take seriously into consideration the praxiology of logical rules.

One therefore recognizes, in Wittgenstein’s view in the NL, a “propositional form.” But already a propositional form quite distinct from the one Frege and Russell supposed was to be recognized in those same acts.

For Russell, this form was only what united, like cement, the various components of the proposition to a subject in an act of judging. Without a subject to unite these terms, the form would not cling to any material support. For Wittgenstein, however, propositional form is sufficient in itself; it is the very possibility of sense, independent of any belief or judgment linked to it.

For Frege, in turn, the form of the proposition is linked to its truth-value. He distinguishes between thought (the content) and judgment (the act of recognizing truth). The assertion sign “ $\vdash$ ” serves to indicate when a proposition is asserted as true. Wittgenstein, however, makes propositional form already contain within itself bipolarity, the capacity to be either true or false, as an immediate datum of symbolic recognition. Thus propositional form in fact defines sense - not truth.

This immediate recognition of what we do in speech or writing is what allows, in the NL, the proposition to be an indicator of reality: “we know what happens if the proposition is true and what happens if it is false” (TS 201a2, p. B4).

But this indication of reality is completely internal. Metaphysical reality did not matter to Wittgenstein in the way it was indispensable to Russell. In a letter to Ottoline Morell, on April 23, 1912, Russell comments:

I argued with him about matter. He thinks that problem is trivial. He admits that if there were no matter, then no one would exist except himself, but that this changes nothing, since physics and astronomy, and all the other sciences, could still be interpreted as true (McGuinness, 1988, p. 106).

Another significant episode in this respect was the discussion between Russell and Wittgenstein about whether or not there was a rhinoceros- or, according to an alternative version by Russell himself, a hippopotamus - in the room at that moment. This took place in the autumn of 1911. For Wittgenstein, the existential proposition was no more important than the asserted proposition, that is, the only one that has sense (cf. McGuinness, 1988, p. 89).

The NL also devote considerable effort to demystifying what Russell and Frege regarded as the most basic components of logic, or, as the text calls them, its “indefinables.” The concept of the “indefinable” is central in the NL to its attempt to establish a “theory of symbolism”:

We must be able to understand propositions that we have never heard before. But every proposition is a new symbol. Therefore, we must have indefinable general symbols; these are inevitable if propositions are not all indefinable (TS 201a2, p. B4).

These indefinables are divided into two main varieties: names (for objects) and forms (for relations). They are indispensable for the apprehension of new sentences. They func-

tion, therefore, as irreducible building blocks that allow language to mirror adequately the structure of the world, without logic itself needing to add any other “objects” or “facts” to the system; without logic itself being consubstantiated into an entity or a “thing” between the world and the speaker.

This principle of economy was, in Potter’s view (cf. 2009, pp. 84–85), an analogy established by Wittgenstein between the indefinables of logic and the unknown elements underlying physical systems, as Hertz admitted them in his *Principles of Mechanics* (1894, pp. 8–9). For Hertz, when we ask what the nature of force is, or what the nature of electricity is, we encounter a far greater lack of answers than when we ask what the nature of gold is, or what the nature of velocity is. Concepts such as “gold” or “velocity” are articulated clearly in our empirical and theoretical representations without giving rise to immediate contradictions; terms such as “force” or “electricity,” however, accumulate ambiguities and poorly reconciled relations that disturb understanding. The most adequate response, for Hertz, is not to try to find new relations and connections through new questions that would only create still more confusion, but to adopt the courageous attitude of *ceasing to ask questions*. This would leave us less tormented and we would cease to propose illegitimate solutions.

It is possible to think of this “principle of economy” as a form of “transcendental argument.” For insofar as we restrict ourselves to limits of possibility, this cannot be done without attending to certain “a priori conditions.” However, adherence to transcendental arguments lies completely outside the scope of Wittgenstein’s philosophical interest. Following strictly Hertz’s pragmatic prescriptions - or “therapeutic” ones, if we wish, since what is at stake is avoiding the disturbances caused by the formulation of illegitimate questions (Hertz, 1894, pp. 8–9) - our author merely proposes the assumption of “operational indefinables” without which we would become mired in dualist empiricism or in some kind of metaphysical realism.

In short, the reader may witness in the NL a pragmatic attitude that clearly takes shape as the prelude to a praxiological philosophy (and one deliberately aesthetic and ethical), which would finally blossom 18 years later, in 1931, and guide his thought from then on until the end of his life.

Notes on Logic

TS 201a2

SUMMARY

III One reason for thinking the old notation wrong is that it is very unlikely that from every proposition p an infinite number of other propositions not-not- p , not-not-not-not- p , etc., should follow.¹

IV If only those signs which contain proper names were complex then propositions containing nothing but apparent variables would be simple. Then what about their denials?

I The verb of a proposition cannot be “is true” or “is false”, but whatever is true or false must already contain the verb.

o- Deductions only proceed according to the laws of deduction, but these laws cannot justify the deduction.

I One reason for supposing that not all propositions which have more than one argument are relational propositions is that if they were, the relations of judgment and inference would have to hold between an arbitrary number of things.

II Every proposition which seems to be about a complex can be analysed into a proposition about its constituents and about the proposition which describes the complex perfectly; i.e., that proposition which is equivalent to saying the complex exists.

I The idea that propositions are names of complexes suggests that whatever is not a proper name is a sign for a relation. Because spatial complexes* consist of Things and Relations only and the idea of a complex is taken from space.

VI In a proposition convert all its indefinables into variables; there then remains a class of propositions which is not all propositions but a type.

VI There are thus two ways in which signs are similar. The names Socrates and Plato are similar: they are both names. But whatever they have in common must not be introduced before Socrates and Plato are introduced. The same applies to a subject-predicate form etc. Therefore, thing, proposition, subject-predicate form, etc., are not indefinables, i.e., types are not indefinables.²

I When we say A judges that etc., then we have to mention a whole proposition which A judges. It will not do either to mention only its constituents, or its constituents and form, but not in the proper order. This shows that a proposition itself must occur in the statement that it is judged; however, for instance, “not- p ” may be explained, the question, “What is negated” must have a meaning.

* Russell – for instance imagines every fact as a spatial complex.

I To understand a proposition p it is not enough to know that p implies “ p is true”, but we must also know that $\sim p$ implies “ p is false”. This shows the bi-polarity of the proposition.

III To every molecular function a WF^* scheme corresponds. Therefore we may use the WF scheme itself instead of the function. Now what the WF scheme does is, it correlates the letters W and F with each proposition. These two letters are the poles of atomic propositions. Then the scheme correlates another W and F to these poles. In this notation all that matters is the correlation of the outside poles to the poles of the atomic propositions. Therefore not-not- p is the same symbol as p . And therefore we shall never get two symbols for the same molecular function.

I The meaning of a proposition is the fact which actually corresponds to it.

III As the ab functions of atomic propositions are bi-polar propositions again we can perform ab operations on them. We shall, by doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions.³

* $W-F$ = Wahr-Falsch.

III The symbolising fact in $a-p-b$ is that, say* a is on the left of p and b on the right of p ; then the correlation of new poles is to be transitive, so that for instance if a new pole a in whatever way i.e. via whatever poles is correlated to the inside a , the symbol is not changed thereby. It is therefore possible to construct all possible ab functions by performing one ab operation repeatedly, and we can therefore talk of all ab functions as of all those functions which can be obtained by performing this ab operation repeatedly.

[Note by Bertrand Russell: ab means the same as WF , which means true-false.]

III Naming is like pointing. A function is like a line dividing points of a plane into right and left ones; then “ p or not- p ” has no meaning because it does not divide the plane.

III But though a particular proposition “ p or not- p ” has no meaning, a general proposition “for all p ’s, p or not- p ” has a meaning because this does not contain the nonsensical function “ p or not- p ” but the function “ p or not- q ” just as “for all x ’s xRx ” contains the function “ xRy ”.⁴

* This is quite arbitrary but, if we once have fixed on which order the poles have to stand we must of course stick to our convention. If for instance “ $a p b$ ” says p then $b p a$ says *nothing*. (It does not say $\sim p$.) But $a - a p b - b$ is the same symbol as $a p b$ (here the ab function vanishes automatically) for here the new poles are related to the same side of p as the old ones. The question is always: how are the new poles correlated to p compared with the way the old poles are correlated to $\sim p$.



TS 201a2, p.A5

I A proposition is a standard⁵ to which facts behave, with names it is otherwise; it is thus bi-polarity and sense comes in; just as one arrow behaves to another arrow by being in the same sense or the opposite, so a fact behaves to a proposition.

II The form of a proposition has meaning in the following way. Consider a symbol “xRy”. To symbols of this form correspond couples of things whose names are respectively “x” and “y”. The things x y stand to one another in all sorts of relations, amongst others some stand in the relation R, and some not; just as I single out a particular thing by a particular name I single out all behaviours of the points x and y with respect to the relation R. I say that if an x stands in the relation R to a y the sign “xRy” is to be called true to the fact and otherwise false. This is a definition of sense.

I In my theory p has the same meaning as not- p but opposite sense. The meaning is the fact. The proper theory of judgment must make it impossible to judge nonsense.⁶

I It is not strictly true to say that we understand a proposition p if we know that p is equivalent to “ p is true” for this would be the case if accidentally both were true or false. What is wanted is the formal equivalence with respect to the forms of the proposition, *i.e.*, all the general indefinables involved. *The sense of an ab function of a proposition is a function of its sense.* There are only unasserted propositions.⁷

TS 201a2, p.A6

I Assertion is merely psychological.⁸ In *not-p*, p is exactly the same as if it stands alone; this point is absolutely fundamental. Among the facts that make “ p or q ” true there are also facts which make “ p and q ” true; if propositions have only meaning, we ought, in such a case, to say that these two propositions are identical, but in fact, their sense is different for we have introduced sense by talking of all p ’s and all q ’s. Consequently the molecular propositions will only be used in cases where their ab function stands under a generality sign or enters into another function such as “I believe that, etc.”, because then the sense enters.

I In “ a judges p ” p cannot be replaced by a proper name. This appears if we substitute “ a judges that p is true and not p is false”. The proposition “ a judges p ” consists of the proper name a , the proposition p with its 2 poles, and a being related to both of these poles in a certain way. This is obviously not a relation in the ordinary sense.⁹

III The ab notation makes it clear that *not* and *or* are dependent on one another and we can therefore not use them as simultaneous indefinables. Same objections in the case of apparent variables to the usual old indefinables, as in the case of molecular functions. The application of the ab notation to apparent variable propositions becomes clear if we consider that, for instance, the proposition “for all x , φx ” is to be true when φx is true for all x ’s and false when φx is false for some x ’s. We see that *some* and *all* occur simultaneously in the proper apparent variable notation.



TS 201a2, p.A7

The Notation is:¹⁰

for $(x) \varphi x : a - (x) - a \varphi x b - (\exists x) - b$ and
 for $(\exists x) \varphi x : a - (\exists x) - a \varphi x b - (x) - b$

Old definitions now become tautologous.¹¹

V In aRb it is not the complex that symbolises but the fact that the symbol a stands in a certain relation to the symbol b . Thus facts are symbolised by facts,¹² or more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world.

I Judgment, question and command are all on the same level. What interests logic in them is only the unasserted proposition. Facts cannot be named.

VI A proposition cannot occur in itself. This is the fundamental truth of the theory of types.¹³

Every proposition that says something indefinable about one thing is a subject-predicate proposition, and so on.

VI Therefore we can recognize a subject-predicate proposition if we know it contains only one name and one form, etc. This gives the construction of types. Hence the type of a proposition can be recognized by its symbol alone.¹⁴

TS 201a2, p.A8

What is essential in a correct apparent-variable notation is this: – (1) it must mention a type of propositions; (2) it must show which components of a proposition of this type are constants.¹⁵

VI [Components are forms and constituents.]

Take (φ) . $\varphi!x$. Then if we describe the *kind* of symbols, for which $\varphi!$ stands and which, by the above, is enough to determine the type, then automatically “ (φ) . $\varphi!x$ ” cannot be fitted by this description, because it *CONTAINS* “ $\varphi!x$ ” and the description is to describe *ALL* that symbolises in symbols of the $\varphi!$ kind. If the description is *thus* complete vicious circles can just as little occur as for instance (φ) . $(X)\varphi$ (where $(X)\varphi$ is a subject-predicate proposition).¹⁶



TS 2012, p.B1

First MS.

Indefinables are of two sorts: names, and forms. Propositions cannot consist of names alone; they cannot be classes of names. A name can not only occur in two different propositions, but can occur in the same way in both.¹⁷

Propositions [which are symbols having reference to facts] are themselves facts: that this inkpot is on this table may express that I sit in this chair.¹⁸

It can never express the common characteristic of two objects that we designate them by the same name but by two different ways of designation, for, since names are arbitrary, we might also choose different names, and where then would be the common element in the designations? Nevertheless one is always tempted, in a difficulty, to take refuge in different ways of designation.¹⁹

Frege said “propositions are names”; Russell said “propositions correspond to complexes”. Both are false; and especially false is the statement “propositions are names of complexes.”²⁰

It is easy to suppose that only such symbols are complex as contain names of objects, and that accordingly “ $(\exists x, \varphi). \varphi x$ ” or “ $(\exists x, y). x R y$ ” must be simple. It is then natural to call the first of these the name of a form, the second the name of a relation. But in that case what is the meaning of (e.g.) “ $\sim(\exists x, y). x R y$ ”? Can we put “not” before a name?²¹

TS 2012, p.B2

The reason why “ $\sim$ Socrates” means nothing is that “ $\sim x$ ” does not express a property of x .

There are positive and negative facts: if the proposition “this rose is not red” is true, then what it signifies is negative. But the occurrence of the word “not” does not indicate this unless we know that the signification of the proposition “this rose is red” (when it is true) is positive. It is only from both, the negation and the negated proposition, that we can conclude to a characteristic of the significance of the whole proposition. (We are not here speaking of negations of *general* propositions, i.e. of such as contain apparent variables. Negative facts only justify the negations of atomic propositions.)

*Positive and negative facts there are, but not true and false facts.*²²

If we overlook the fact that propositions have a *sense* which is independent of their truth or falsehood, it easily seems as if true and false were two equally justified relations between the sign and what is signified. (We might then say e.g. that “ q ” *signifies* in the true way what “not- q ” *signifies* in the false way). But are not true and false in fact equally justified? Could we not express ourselves by means of false propositions just as well as hitherto with true ones, so long as we know that they are meant falsely? No! For a proposition is then true when



TS 2012, p.B3

it is as we assert in this proposition; and accordingly if by “ q ” we mean “not- q ”, and it is as we mean to assert, then in the new interpretation “ q ” is actually true and *not* false. But it is important that we *can* mean the same by “ q ” as by “not- q ”, for it shows that neither to the symbol “not” nor to the manner of its combination with “ q ” does a characteristic of the denotation of “ q ” correspond.

TS 2012, p.B4

Second MS.

We must be able to understand propositions which we have never heard before. But every proposition is a new symbol. Hence we must have *general* indefinable symbols; these are unavoidable if propositions are not all indefinable.²³

Whatever corresponds in reality to compound propositions must not be more than what corresponds to their several atomic propositions.²⁴

Not only must logic not deal with [particular] things, but just as little with relations and predicates.²⁵

There are no propositions containing real variables.²⁶

What corresponds in reality to a proposition depends upon whether it is true or false. But we must be able to understand a proposition without knowing if it is true or false.

What we know when we understand a proposition is this: We know what is the case if the proposition is true, and what is the case if it is false. But we do not know [necessarily] whether it is true or false.²⁷

Propositions are not names.

We can never distinguish one logical type from another by attributing a property to members of the one which we deny to members of the other.²⁸

TS 2012, p.B5

Symbols are not what they seem to be. In “ $a R b$ ”, “ R ” looks like a substantive, but is not one. What symbolizes in “ $a R b$ ” is that R occurs between a and b . Hence “ R ” is *not* the indefinable in “ $a R b$ ”. Similarly in “ φx ”, “ φ ” looks like a substantive but is not one; in “ $\sim p$ ”, “ $\sim$ ” looks like “ φ ” but is not like it. This is the first thing that indicates that there *may* not be logical constants. A reason against them is the generality of logic: logic cannot treat a special set of things.

Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms.

All that is essential about molecular functions is their T-F schema [i.e. the statement of the cases when they are true and the cases when they are false].

Alternative indefinability shows that the indefinables have not been reached.²⁹

Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two



poles, corresponding to the case of its truth and the case of its falsehood. We call this the *sense* of a proposition.

TS 201a2, p.B6

v In regard to notation, it is important to note that not every feature of a symbol symbolizes. In two molecular functions which have the same T-F schema, what symbolizes must be the same. In “not-not-*p*”, “not-*p*” does not occur; for “not-not-*p*” is the same as “*p*”, and therefore, if “not-*p*” occurred in “not-not-*p*”, it would occur in “*p*”.

iii Logical indefinables cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations. Nor are “not” and “or”, like judgment, *analogous* to predicates or relations, because they do not introduce anything new.³⁰

iv Propositions are always complex even if they contain no names.³¹

ii A proposition must be understood when *all* its indefinables are understood. The indefinables in “a R b” are introduced as follows:

“*a*” is indefinable;

“*b*” is indefinable;

Whatever “x” and “y” may mean, “x R y” says something indefinable about their meaning.

v A complex symbol must never be introduced as a single indefinable. [Thus e.g. no proposition is indefinable]. For if one of its parts occurs also in another connection, it must there be re-introduced. And would it then mean the same?

TS 201a2, p.B7

The ways by which we introduce our indefinables must permit us to construct all propositions that have sense [? meaning] from these indefinables *alone*. It is easy to introduce “all” and “some” in a way that will make the construction of (say) “(x, y). x R y” possible from “all” and “x R y” *as introduced before*.

TS 201a2, p.B8

3rd. MS.

i An analogy for the theory of truth: Consider a black patch on white paper; then we can describe the form of the patch by mentioning, for each point of the surface, whether it is white or black. To the fact that a point is black corresponds a positive fact, to the fact that a point is white (not black) corresponds a negative fact. If I designate a point of the surface (one of Frege’s “truth-values”), this is as if I set up an assumption to be decided upon. But in order to be able to say of a point that it is black or that it is white, I must first know when a point is to be called black and when it is to be called white. In order to be able to say that “*p*” is true (or false), I must first have determined under what circumstances I call a proposition true, and thereby I



determine the *sense* of a proposition. The point in which the analogy fails is this: I can indicate a point of the paper what is white and black, but to a proposition without sense nothing corresponds, for it does not designate a thing (truth-value), whose properties might be called “false” or “true”; the verb of a proposition is not “is true” or “is false”, as Frege believes, but what is true must already contain the verb.³²

TS 201a2, p.B9

i The comparison of language and reality is like that of retinal image and visual image: to the blind spot nothing in the visual image seems to correspond, and thereby the boundaries of the blind spot determine the visual image – as true negations of atomic propositions determine reality.³³

iii Logical inferences can, it is true, be made in accordance with Frege’s or Russell’s laws of deduction, but this cannot justify the inference; and therefore they are not primitive propositions of logic. If *p* follows from *q*, it can also be inferred from *q*, and the “manner of deduction” is indifferent.³⁴

iv Those symbols which are called propositions in which “variables occur” are in reality not propositions at all, but only schemes of propositions, which only become propositions when we replace the variables by constants. There is no proposition which is expressed by “x = x”, for “x” has no signification; but there is a proposition “(x). x = x” and propositions such as “Socrates = Socrates” etc.³⁵

iv In books on logic, no variables ought to occur, but only the general propositions which justify the use of variables. It follows that the so-called definitions of logic are not definitions, but only schemes of definitions, and instead of these we ought to put general propositions; and similarly the so-called primitive ideas (Urzeichen) of logic are not primitive ideas, but the schemes of them. The mistaken idea that there

TS 201a2, p.B10

are things called facts or complexes and relations easily leads to the opinion that there must be a relation of questioning(?) to the facts, and then the question arises whether a relation can hold between an arbitrary number of things, since a fact can follow from arbitrary cases. It is a fact that the proposition which e.g. expresses that *q* follows from *p* and $p \supset q$ is this: $p. p \supset q. \supset p.q.q$.

i At a pinch, one is tempted to interpret “not-*p*” as “everything else, only not *p*”. That from a single fact *p* an infinity of others, not-not-*p* etc., follow, is hardly credible. Man possesses an innate capacity for constructing symbols with which *some* sense can be expressed, without having the slightest idea what each word signifies. The best example of this is mathematics, for man has until lately used the symbols for numbers without knowing what they signify or that they signify nothing.

ii Russell’s “complexes” were to have the useful property of being compounded, and were to combine with this the agreeable property that they could be treated like “simples”. But this alone made them unserviceable as logical types, since there would have been significance in asserting, of a simple, that it was complex. But a *property* cannot be a logical type.³⁶



TS 2012a, p.B11

II Every statement about apparent complexes can be resolved into the logical sum of a statement about the constituents and a statement about the proposition which describes the complex completely. How, in each case, the resolution is to be made, is an important question, but its answer is not unconditionally necessary for the construction of logic.

III That “or” and “not” etc. are not relations in the same sense as “right” and “left” etc., is obvious to the plain man. The possibility of cross-definitions in the old logical indefinables shows, of itself, that these are not the right indefinables, and, even more conclusively, that they do not denote relations.³⁷

VI If we change a constituent a of a proposition $\varphi(a)$ into a variable, then there is a class

$$\hat{p} \{(\exists x). \varphi(x) = p\}.$$

This class in general still depends upon what, by an *arbitrary convention*, we mean by “ $\varphi(x)$ ”. But if we change into variables all those symbols whose significance was arbitrarily determined, there is still such a class. But this is now not dependent upon any convention, but only upon the nature of the symbol “ $\varphi(x)$ ”. It corresponds to a logical type.³⁸

TS 2012a, p.B12

VI Types can never be distinguished from each other by saying (as is often done) that one has these *but* the other has those properties, for this presupposes that there is a *meaning* in asserting all these properties of both types. But from this it follows that, at best, these properties may be types, but certainly not the objects of which they are asserted.

I At a pinch we are always inclined to explanations of logical functions of propositions which aim at introducing into the function either only the constituents of these propositions, or only their form, etc. etc.; and we overlook that ordinary language would not contain the whole propositions if it did not need them: However, e.g., “not- p ” may be explained, there must always be a meaning given to the question “what is denied?”

III The very possibility of Frege’s explanations of “not- p ” and “if p then q ”, from which it follows that “not-not- p ” denotes the same as p , makes it probable that there is some method of designation in which “not-not- p ” corresponds to the same symbol as “ p ”. But if this method of designation suffices for logic, it must be the right one.

I Names are points, propositions arrows – they have *sense*. The sense of a proposition is determined by the two poles *true* and *false*. The form of a proposition is like a straight line, which divides all points of a plane into right and left. The line does this automatically, the form of proposition only by convention.



TS 2012a, p.B13

II Just as little as we are concerned, in logic, with the relation of a name to its meaning, just so little are we concerned with the relation of a proposition to reality, but we want to know the meaning of names and the sense of propositions as we introduce an indefinable concept “A” by saying: “‘A’ denotes something indefinable”, so we introduce e.g. the form of propositions $a R b$ by saying: “For all meanings of “x” and “y”, “x R y” expresses something indefinable about x and y”.

III In place of every proposition “p”, let us write “ $\frac{a}{b} p$ ”. Let every correlation of propositions to each other or of names to propositions be effected by a correlation of their poles “a” and “b”. Let this correlation be transitive. Then accordingly “ $\frac{a}{b} - \frac{a}{b} p$ ” is the same symbol as “ $\frac{a}{b} p$ ”. Let n propositions be given. I then call a “class of poles” of these propositions every class of n members, of which each is a pole of one of the n propositions, so that one member corresponds to each proposition. I then correlate with each class of poles one of two poles (a and b). The sense of the symbolizing fact thus constructed I cannot define, but I know it.

III If $p = \text{not-not-}p$ etc., this shows that the traditional method of symbolism is wrong, since it allows a plurality of symbols with the same sense; and thence it follows that, in analyzing such propositions, we must not be guided by Russell’s method of symbolizing.³⁹

TS 2012a, p.B14

V It is to be remembered that names are not things, but classes: “A” is the same letter as “A”. This has the most important consequences for every symbolic language.⁴⁰

I Neither the sense nor the meaning of a proposition is a thing. These words are incomplete symbols.

V It is impossible to dispense with propositions in which the same argument occurs in different positions. It is obviously useless to replace $\varphi(a, a)$ by $\varphi(a, b)$. $a = b$.

III Since the ab -functions of p are again bi-polar propositions, we can form ab -functions of them, and so on. In this way a series of propositions will arise, in which in general the *symbolizing* facts will be the same in several members. If now we find an ab -function of such a kind that by repeated application of it every ab -function can be generated, then we can introduce the totality of ab -functions as the totality of those that are generated by application of this function. Such a function is $\sim p \vee \sim q$.

III It is easy to suppose a contradiction in the fact that on the one hand every possible complex proposition is a simple ab -function of simple propositions, and that on the other hand the repeated application of one ab -function suffices to generate all these propositions. If e.g. an affirmation can be generated by double negation, is negation in any sense contained in affirmation? Does “p” deny “not-p” or assert “p”, or both? And how do

TS 2012a, p.B15

matters stand with the definition of “ $\supset$ ” by “ $\vee$ ” and “ $\cdot$ ”, or of “ $\vee$ ” by “ $\cdot$ ” and “ $\supset$ ”? And how e.g. shall we introduce $p|q$ (i.e. $\sim p \vee \sim q$), if not by saying that this expression says something indefinable about all arguments p and q ? But the ab -functions must be introduced as follows: The



function $p|q$ is merely a mechanical instrument for constructing all possible *symbols* of *ab*-functions. The symbols arising by repeated application of the symbol “|” do *not* contain the symbol “ $p|q$ ”. We need a rule according to which we can form all symbols of *ab*-functions, in order to be able to speak of the class of them; and we now speak of them e.g. as those symbols of functions which can be generated by repeated application of the operation “|”. And we say now: For all p ’s and q ’s, “ $p|q$ ” says something indefinable about the sense of those simple propositions which are contained in p and q .

I The assertion-sign is logically quite without significance. It only shows, in Frege and Whitehead and Russell, that these authors hold the propositions so indicated to be true. “ $\vdash$ ” therefore belongs as little to the proposition as (say) the number of the proposition. A proposition cannot possibly assert of itself that it is true.

Every right theory of judgment must make it impossible for me to judge that this table penholders the book. Russell’s theory does not satisfy this requirement.⁴¹

TS 201a2, p.B16

I It is clear that we understand propositions without knowing whether they are true or false. But we can only know the *meaning* of a proposition when we know if it is true or false. What we understand is the *sense* of the proposition.

III The assumption of the existence of logical objects makes it appear remarkable that in the sciences propositions of the form “ $p \vee q$ ”, “ $p \supset q$ ”, etc. are only then not provisional when “ $\vee$ ” and “ $\supset$ ” stand within the scope of a generality-sign [apparent variable].⁴²

TS 201a2, p.B17

4th. MS.

II If we formed all possible atomic propositions, the world would be completely described if we declared the truth or falsehood of each. [I doubt this.]⁴³

I The chief characteristic of my theory⁴⁴ is that, in it, p has the same *meaning* as not- p .

II A false theory of relations makes it easily seem as if the relation of fact and constituent were the same as that of fact and fact which follows from it. But the similarity of the two may be expressed thus: $\varphi a . \supset_{\varphi, a} . a = a$.⁴⁵

II If a word creates a world so that in it the principles of logic are true, it thereby creates a world in which the whole of mathematics holds; and similarly it could not create a world in which a proposition was true, without creating its constituents.⁴⁶

III Signs of the form “ $p \vee \sim p$ ” are senseless, but not the proposition “ $(p) . p \vee \sim p$ ”. If I know that this rose is either red or not red, I know nothing. The same holds of all *ab*-functions.⁴⁷

I To understand a proposition means to know what is the case if it is true. Hence we can understand it without knowing if it is true. We understand it when we understand its constituents and forms. If we know the meaning of “ a ” and “ b ”, and if we know what “ $x R y$ ” means for all x ’s and y ’s, then we also understand “ $a R b$ ”.



TS 201a2, p.B18

I I understand the proposition “ $a R b$ ” when I know that either the fact that $a R b$ or the fact that not $a R b$ corresponds to it; but this is not to be confused with the false opinion that I understood “ $a R b$ ” when I know that “ $a R b$ or not $a R b$ ” is the case.⁴⁸

II But the form of a proposition symbolizes in the following way: Let us consider symbols of the form “ $x R y$ ”; to these correspond primarily pairs of objects, of which one has the name “ x ”, the other the name “ y ”. The x ’s and y ’s stand in various relations to each other, among others the relation R holds between some, but not between others. I now determine the sense of “ $x R y$ ” by laying down: when the facts behave in regard to “ $x R y$ ” so that the meaning of “ x ” stands in the relation R to the meaning of “ y ”, then I say that they [the facts] are “of like sense” [“gleichsinnig”] with the proposition “ $x R y$ ”; otherwise, “of opposite sense” [“entgegengesetzt”]; I correlate the facts to the symbol “ $x R y$ ” *by* thus dividing them into those of like sense and those of opposite sense. To this correlation corresponds the correlation of name and meaning. Both are psychological. Thus I understand the form “ $x R y$ ” when I know that it discriminates the behaviour of x and y according as these stand in the relation R or not. In this way I extract from all possible relations the relation R , as, by a name, I extract its meaning from among all possible things.

TS 201a2, p.B19

I Strictly speaking, it is incorrect to say: we understand the proposition p when we know that “ p is true” $\equiv p$; for this would naturally always be the case if accidentally the propositions to right and left of the symbol “ $\equiv$ ” were both true or both false. We require not only an equivalence, but a formal equivalence, which is bound up with the introduction of the form of p .

III The sense of an *ab*-function of p is a function of the sense of p .

III The *ab*-functions use the discrimination of facts, which their arguments bring forth, in order to generate new discriminations.

V Only facts can express sense, a class of names cannot. This is easily shown.

II There is no thing which is the form of a proposition, and no name which is the name of a form. Accordingly we can also not say that a relation which in certain cases holds between things holds sometimes between forms and things. This goes against Russell’s theory of judgment.⁴⁹

~~It is very easy to forget that, though the propositions of a form can be either true or false, each one of these propositions can only be either true or false, not both.~~

TS 201a2, p.B20

III Among the facts which make “ p or q ” true, there are some which make “ p and q ” true; but the class which makes “ p or q ” true is different from the class which makes “ p and q ” true; and only this is what matters. For we introduce this class, as it were, when we introduce *ab*-functions.

IV A very natural objection to the way in which I have introduced e.g. propositions of the form $x R y$ is that by it propositions such as $(\exists x . y) . x R y$ and similar ones are not explained, which yet obviously have in common with $a R b$ what $c R d$ has in common with $a R b$. But when



we introduce propositions of the form $x R y$ we mentioned no one particular proposition of this form; and we only need to introduce $(\exists x, y). \varphi(x, y)$ for all φ 's in any way which makes the sense of these propositions dependent on the sense of all propositions of the form $\varphi(a, b)$, and thereby the justness of our procedure is proved.

V The indefinables of logic must be independent of each other. If an indefinable is introduced, it must be introduced in all combinations in which it can occur. We cannot therefore introduce it first for one combination, then for another; e.g., if the form $x R y$ has been introduced, it must henceforth be understood in propositions of the form $a R b$ just in the same way as in propositions such as $(\exists x, y). x R y$ and others. We must not introduce it first for one class of cases, then for the other; for it would remain doubtful if its meaning was the same in

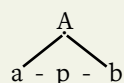
TS 201a2, p.B21

both cases, and there would be no ground for using the same manner of combining symbols in both cases. In short, for the introduction of indefinable symbols and combinations of symbols the same holds, *mutatis mutandis*, that Frege has said for the introduction of symbols by definitions.

III It is *a priori* likely that the introduction of atomic propositions is fundamental for the understanding of all other kinds of propositions. In fact the understanding of general propositions obviously depends on that of atomic propositions.

IV Cross-definability in the realm of general propositions leads to quite similar questions to those in the realm of *ab*-functions.

I When we say "A believes *p*", this sounds, it is true, as if here we could substitute a proper name for "*p*"; but we can see that here a *sense*, not a meaning, is concerned, if we say "A believes that '*p*' is true"; and in order to make the direction of *p* even more explicit, we might say "A believes that '*p*' is true and 'not-*p*' is false". Here the bi-polarity of *p* is expressed and it seems that we shall only be able to express the proposition "A believes *p*" correctly by the *ab*-notation; say by making "A" have a relation to the poles "a" and "b" of a-*p*-b.



The epistemological questions concerning the nature of judgment and belief cannot be solved without a correct apprehension of the form of the proposition.⁵⁰

TS 201a2, p.B22

III The *ab*-notation shows the dependence of *or* and *not*, and thereby that they are not to be employed as simultaneous indefinables.

V *Not*: "The complex sign '*a R b*'" says that *a* stands in the relation *R* to *b*; but *that* '*a*' stands in a certain relation to '*b*' says *that* *a R b*.

In philosophy there are no deductions: *it* is purely descriptive.

Philosophy gives no pictures of reality.

Philosophy can neither confirm nor confute scientific investigation.

Philosophy consists of logic and metaphysics: logic is its basis.

Epistemology is the philosophy of psychology.

Distrust of grammar is the first requisite for philosophizing.⁵¹

P r e -
l i m i -
n a r y

~~Propositions can never be indefinables, for they are always complex. That also words like "ambulo" are complex appears in the fact that their root with a different termination gives a different sense.~~

II Only the doctrine of general indefinables permits us to understand the nature of functions. Neglect of this doctrine leads to an impenetrable thicket.

TS 201a2, p.B23

P r e - l i m i - n a r y Philosophy is the doctrine of the logical form of scientific propositions (not only of primitive propositions).

The word "philosophy" ought always to designate something over or under but not beside, the natural sciences.

I Judgment, command and question all stand on the same level; but all have in common the propositional form, which does interest *us*.

I The structure of the proposition must be recognized, the rest comes of itself. But ordinary language conceals the structure of the proposition: in it, relations look like predicates, predicates like names, etc.

I Facts cannot be *named*.

VI It is easy to suppose that "individual", "particular", "complex" etc. are primitive ideas of logic. Russell e.g. says "individual" and "matrix" are "primitive ideas". This error presumably is to be explained by the fact that, by employment of variables instead of the generality-sign, it comes to seem as if logic dealt with things which have been deprived of all properties except thing-hood, and with propositions deprived of all properties except complexity. We forget that the indefinables of symbols [*Urbilder von Zeichen*] only occur under the generality-sign, never outside it.

TS 201a2, p.B24

IV Just as people used to struggle to bring all propositions into the subject-predicate form, so now it is natural to conceive every proposition as expressing a relation, which is just as incorrect. What is justified in this desire is fully satisfied by Russell's theory of manufactured relations.⁵²

I One of the most natural attempts at solution consists in regarding "not-*p*" as "the opposite of *p*", where then "opposite" would be the indefinable relation. But it is easy to see that every such attempt to replace the *ab*-functions by descriptions must fail.

I The false assumption that propositions are names leads us to believe that there must be logical objects: for the meanings of logical propositions will have to be such things.

P r e - l i m i - n a r y A correct explanation of logical propositions must give them a unique position as against all other propositions.

VI No proposition can say anything about itself, because the symbol of the proposition cannot be contained in itself; this must be the basis of the theory of logical types.

VI Every proposition which says something indefinable about a thing is a subject-predicate proposition; every proposition which says something indefinable about two things expresses a dual relation between these things, and so on. Thus every proposition which contains only one name and one indefinable form is a subject-predicate proposition, and so on. An



TS 201a2, p.B35

indefinable simple symbol can only be a name, and therefore we can know, by the symbol of an atomic proposition, whether it is a subject-predicate proposition.

TS 201a2, p.B36

Wittg.-

I. Bi-polarity of propositions: sense & meaning, truth & falsehood.

II. Analysis of atomic propositions: general indefinables, predicates, etc.

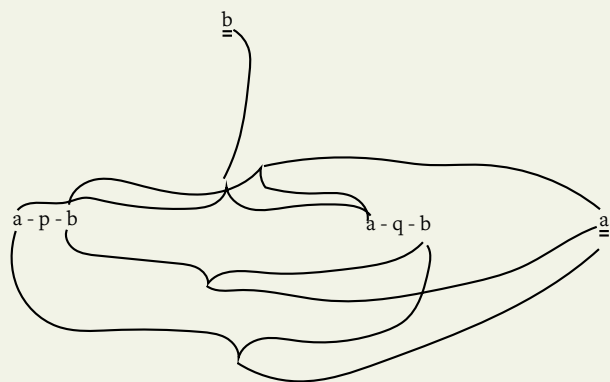
III. Analysis of molecular propositions: ab-functions.

IV. Analysis of general propositions.

V. Principles of symbolism: what symbolizes in a symbol. Facts for facts.

VI. Types.⁵³

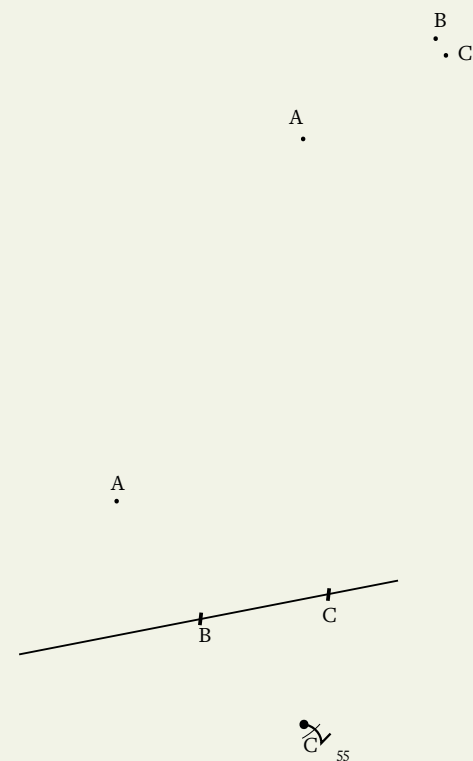
TS 201a2, p.B37



This is the symbol for $\sim p \vee \sim q$ ⁵⁴



TS 201a2, p.B37v



COMMENTED NOTES

The abbreviations used in this section are:

- [SUM] - for notes from the Summary;
- [MS1] - for notes from MS1;
- [MS2] - for notes from MS2;
- [MS3] - for notes from MS3;
- [MS4] - for notes from MS4.

1. [SUM] In this paragraph Wittgenstein attacks the idea that logical operations, such as negation, add content or objects to reality. For Russell and Frege, with whom our philosopher is particularly concerned, logical connectives were treated almost like mathematical functions that produce new logical objects. But if negation were a real “constituent” of the proposition, as these authors suppose, then every time we added a “not”, we would be constructing a new complex structure. For Wittgenstein, logical signs represent nothing. There is no “object” called negation in the world. Negation is merely an operation of inversion. If we invert something and then invert it again, we return to the original point. p and $\sim\sim p$ are not two propositions where one follows from the other; at bottom, they are the same symbol viewed from angles that cancel each other out. Operations such as double negation must “disappear” in the final analysis, rather than accumulate infinitely. The criticism here opposes the reification of logical symbols.
2. [SUM] Here Wittgenstein draws a distinction between *indefinables* and *types* that is clearly at odds with Russell’s distinction between these categories. We may say that he is laying the groundwork for what would later become his critique of the logical hierarchies of his mentor and, up to that point, friend. For Wittgenstein, indefinables are basic symbols that have meaning by themselves within a proposition. In the example cited, “Socrates” and “Plato” function as names that refer to objects. They are *starting points*. We cannot explain what “Socrates” is without first pointing to the entity and introducing the name. They are primitive in the sense that logic deals with them as the basic building blocks of representation. A “type”, by contrast, is not something that exists independently as one of these blocks; it is only a resemblance in the way signs function. Names always function in the same way, but we only notice this after they have already been introduced and used in propositions. Indefinables are the pieces of the game, whereas types are merely the categories that describe how those pieces look or move. We do not “define” types; we recognize them in the structure of the proposition.
3. [SUM] What is proposed in this paragraph corresponds to the chart presented at the end, on page TS 201a, p. B27r, where we visualize the combinations of truth-values (T or F) of the proposition $\sim p \vee \sim q$. This combination, which today is usually made by means of a table, Wittgenstein initially imagined through braces in which each atomic proposition (p or q) already appears accompanied by its two truth-values a and b . Thus, $a - p - b$ and $a - q - b$. The braces connect them in successive outer layers, until, in the outermost layers, we can see which combinations are true and false. In the case of the proposition under discussion, it is b only when both p and q are a ; in all other cases it is a . In this way, we visualize the truth-values in all cases of $\sim p \vee \sim q$. It should be noted, in passing, that this operation is eminently recursive. It does not depend on any prior semantic

assumption.

4. [SUM] In this passage, Wittgenstein is trying to define what distinguishes a name from a proposition, and why tautologies (such as “ p or not- p ”) occupy a strange place in the system. If “naming is like pointing”, then the name, by itself, says absolutely nothing about the world. It is there only to occupy the place of the object. If the name does not find a corresponding object, it has no reference, but it does not “miss” the target; it simply does not function. Here, however, lies the brilliance of the metaphor: let us imagine an infinite plane of possibilities, like “the world”; then a proposition (a logical function) acts as a line that cuts across that plane. If we draw a straight line, we create the following distinction: on one side are the facts that would make the proposition true, and on the other, the facts that would make it false. In the proposition “It is raining”, the left side, for example, is the scenario in which water falls from the sky (true); the right side includes all the other scenarios in which it does not rain (false). A proposition has sense only if it can be true but can also be false. It must “risk” an assertion about how the world is, excluding all other possibilities. If it can be neither true nor false, as in “ p or not- p ”, it does not divide the plane and excludes absolutely nothing. If there is no point to be excluded by an assertion, there is no information in it. This is the case with the tautology “It is raining or it is not raining”. Notice how Wittgenstein is imploding Frege’s and Russell’s definitions of what thought and logic are. While his masters saw logic as a science of objects or universal laws, Wittgenstein sees it as a geometry of representation. Against Frege, thought here is not an “object”. In Frege’s view, a proposition was the name of an abstract object: the true or the false. Thus, “naming” and “asserting” were almost the same operation. For Wittgenstein, a name merely points (it has no truth-value by itself), whereas a proposition divides the plane. Against Russell, Wittgenstein maintains that logic has no content. Whereas for Russell a tautology such as “ p or not- p ” is a supreme truth about the world, for Wittgenstein logic does not deal with “facts”, but with the “form of facts”. While Frege and Russell sought the “bricks” of reality (objects, functions, relations), Wittgenstein is interested in the space in which those bricks are placed. To understand a proposition is not to know that it is true, but to know what occurs if it is true and what occurs if it is false. Frege and Russell did not accept this “bipolarity” as the essence of sense; for them, the focus was on the assertion of truth. Wittgenstein shifts the focus to the possibility of dividing the plane.
5. [SUM] The text says: “a proposition is a pattern according to which facts behave”. This is what our author will call, in the following paragraph, the “form of a proposition”. This is also the set of rules according to which our speech, or our writing, is expressed in symbolism. Of course, ordinary language may have, according to the author’s understanding at this inaugural moment of his thought, a “bad symbolism”. This happens when the same signs are used for different syntactic functions. We use, for example, the verb “to be” both for the attribution of a property and to express identity. We say “John is tall”, but we also say “John is the teacher”. The example that appears in the text is that of a relation: “ xRy ”. This propositional form expresses that two things, x and y , are placed, in relation to one another, in a certain kind of relation. The fact referred to may or may not obey this pattern, and the proposition will therefore, by its very form, be either true or false. But is this “pattern”, this “form of a proposition”, a kind of “transcendental argument”, that is, the defense of a condition of possibility according to which we recognize facts? In TS 201a, p. B24, it is stated that “The structure of the proposition has to be recognized; the rest follows of itself”. The problem seems to arise from the question: who recognizes it. Is it a consciousness, and in that case a “transcendental consciousness”; or a norm, and in that case a “transcendental social rule”? Wittgenstein, however, simply points to ordinary language, which “conceals the structure of the proposition”. This cannot mean “transcendental consciousness”, of course, since no representation of representations is indicated that would gather representational multiplicity

into a certain orientation or subsume it under rules; nor can it suggest a normative instance, because in that case we would have as many logics as there are social conventions. What we have in the text, therefore, are not properly “transcendental arguments”, but simply - or merely - *speech acts*, or written texts, which carry with them, opaquely, the forms of the propositions that can be recognized. This whole varied set of forms, associated with indefinables, generally shapes what is called “symbolism”. This is expressed in speech or written language. Recognition is in symbolism; it is part of it, but it does not create the structure: it only makes it visible. Symbolism obeys the articulations of logical syntax. So, yes, perhaps we can say that recognition is an operation made possible, at bottom, by a set of articulations that structure language. But if it is a condition of possibility, it is neither epistemic, nor normative, nor communicational. This set of fitting rules, logical syntax, which allows symbolism to function as expression in speech and writing, which allows, let us say, thought, is not, according to the text, inside the world or inside the human mind. Symbolism allows us to recognize the structure because it imposes limits; limits of sense on what can be spoken and written, and on what can be thought. When we write “ $\varphi(a)$ ”, the “function-argument” structure leaps to the eye. But one can take, on the other hand, symbolism as a transcendental condition of possibility without any foundation, since this capacity allows us to think, but does not allow the philosophical structure that we conventionally call a “transcendental argument”. In the case of the NL, symbolism simply shows itself. It is the recognizable as well as the recognizing manifestation of syntax. But the pattern operates in a semantic void; it operates by itself, like bones that make possible the movement of the body of language. We know only its rules because we speak and think, and our signs, if they have sense, inexorably obey the form of the proposition. But ultimately, we have only the spoken and written expression through which we recognize a universal logical syntax. The transcendental argument, even if its formulation in philosophical terms is possible, simply does not belong to logic and it is of no interest to Wittgenstein. And this, it seems to me, is just a pragmatic decision.

6. [SUM] Here we have a lethal challenge to Russell’s theory of judgment. Russell tried to explain judgment (the act of judging, of formulating a judgment), as a relation between a subject and several objects, as in “J (S, A, R, B): John judges that A loves B”. For Russell, the elements S (the subject “John”), the relation R (“love”), and its components A and B were loose bricks that the subject’s mind joined together in an act of judgment. But this formulation did not prevent a subject from formulating a judgment about total nonsense, such as “John judges that the pen loves Wednesday”, or “This table pen-holders the book” (TS 201a, p. B14). For Wittgenstein, a judgment is not a relation between a mind and loose objects, but the perception, elaboration, and utterance of a *sense*. Therefore, we do not need a mental faculty to tell us what is possible or impossible; logical syntax itself must prevent the formation of senseless combinations. A correct theory of judgment must show us that we can formulate judgments only about something that already possesses, in advance, a bipolar sense; that is, something that can be true or false. To judge nonsense is not a factual error; it is a category error: it is to try to point to a place that does not exist on the logical map. By the same understanding, we shall see later that a judgment is not analogous to predicates or relations, since it introduces nothing new (TS 201a, p. B6). Wittgenstein will also add that “Judgment, command and question all stand on the same level (...)” (TS 201a, p. A7; TS 201a, p. B23). That is, he proposes that, from the point of view of logic, what matters is only the form of the possibility that the proposition describes, not the speech act (the command or the question), nor psychology (the judgment or belief). This is also, it should be added, a completely pragmatic orientation of logic: a reminder of how we should apply its rules, or of how it makes sense to apply them if we take them as a paradigm.

7. [SUM] This paragraph can be divided into three parts if we want to understand clearly what Wittgen-

stein is saying here: (1) the first concerns the difference between accidental equivalence and formal equivalence; (2) the second concerns the “*ab* function”; and (3) the third concerns “unasserted propositions”. As for (1), the paragraph says: “It is not strictly true to say that we understand a proposition *p* if we know that *p* is equivalent to ‘*p* is true’, for this would be the case if, accidentally, both were true or false”. Our author is here combating an idea that seemed obvious at the time - defended by logicians such as Gottlob Frege - namely, that the sense of a sentence *p* is simply to know under what conditions it is true. He argues that if you merely say that *p* and “*p* is true” have the same value (both false or both true), this may be a mere coincidence of worldly facts, an accidental matter. To understand a proposition goes far beyond knowing whether it matches current reality; understanding means grasping its deep logical structure, its formal equivalence, and how its fundamental components (“general indefinables”) are related. We understand a proposition only if we know what would make it true and what would make it false. As for (2), the paragraph says: “The sense of an *ab* function of a proposition is a function of its sense”. Remembering that in the *ab* function *a* represents the true pole and *b* represents the false pole, when our author says that the sense of an *ab* function is a function of the sense of the original proposition, he means that logic is purely combinatorial. If you know the sense of *p* and the sense of *q*, the sense of “*p* & *q*” depends solely and exclusively on the combination of the senses of *p* and *q*. Logic adds no new information about the world; it merely rearranges the possibilities that were already present in the original propositions. As for (3), to say that “there are only unasserted propositions” means to oppose Frege’s and Russell’s idea that logic needs a sign of assertion in order to say “This here is a fact!”. Wittgenstein is saying that, in truth, logic does not care about what is actually true or false in the real world. A proposition, by itself, is only a proposal of how things could be in the world. It projects a possibility. If I say “It is raining”, the proposition has exactly the same sense whether it is in fact raining, whether it is a lie, or whether it occurs within a hypothesis such as “If it is raining, I shall take the umbrella” (where no one is asserting that it is raining). For logic, the proposition is neutral: it is always “unasserted”. The truth-value, whether it is a fact or not, is a problem of the world, not of the structure of language.

8. [SUM] It is curious, to say the least, that Wittgenstein says, in a context in which logic is constituted precisely in opposition to psychology, that “assertion is merely psychological”. At first we might think that the young author’s mentors, Frege and Russell, are here receiving a direct blow. For these two authors, when we affirm something such as “Snow is white”, we place in the sentence a special force: an “assertion sign” (I-) that says: “This here is a truth!”. To understand why, we need to dissect the two major arguments he presents in this passage: (1) the argument from negation; and (2) the argument from molecular functions. Taking (1), Wittgenstein notices a serious logical problem here. Consider the proposition: “It is not true that snow is white” ($\sim p$). If the original *p* contained within itself the force of being a true assertion, then when you placed “Not” before it, you would be denying an already established truth, generating a logical short circuit. For logic to work, the content of the proposition *p* (“snow is white”) must be exactly the same when it stands alone, when it is negated, or when it appears within a hypothesis (“If snow is white”). The act of “asserting with conviction” is merely a mental state of the speaker, a psychological act of belief. Logic, by contrast, cares only about the neutral structure of the proposition, which expresses a possibility that may be true or false. As for (2), Wittgenstein explains the crucial difference between what he calls “meaning” (which here carries the weight of truth-value or whether something is true or false in the world) and “sense” (the logical direction of the proposition). A little further down in the paragraph, he says: “Among the facts that make ‘*p* or *q*’ true, there are also facts that make ‘*p* and *q*’ true; if propositions had only meaning, we should in this case have to say that these two propositions are identical”. Imagine, for example, that it is raining (*p*) and that it is cold (*q*);

in the real world, the fact that “it is raining and it is cold” makes the sentence $p \& q$ true; the very same fact also makes the sentence $p \vee q$ true. If factual truth-value - the “meaning” - were the only thing that mattered, the two propositions would be identical in that situation, for both point to the same true result. But we know that they are not the same. Their sense is different because they cover different possibilities (“all the ps and all the qs ”). Taking (1) and (2) together, Wittgenstein concludes with the rule governing the use of molecular propositions: “...Consequently, molecular propositions will only be used in cases where their ab function is under a sign of generality or forms part of another function such as ‘I believe that etc.’, because then the sense is part of it”. If the real world has already settled what is true or false - if the facts are already given - we would not need complex logic (“molecular propositions” such as $p \vee q$) to describe bare reality. It would be enough to point to isolated facts. We use these logical combinations only in two scenarios where sense, i.e., the map of possibilities, matters: (a) under a sign of generality, when we formulate general logical or scientific laws (e.g., “For every x , if x is a human being, x is mortal”) - we are not speaking of a specific fact, but of a structure of possibilities; and (b) in psychological contexts (“I believe that...”, “I think that...”); when I say “John believes that $p \vee q$ ”, what is in John’s mind is not the accomplished fact in the world, but rather the sense of the proposition; he is evaluating logical possibilities. That is why, in these cases, the sense “is part” of the function, since human psychology operates upon possibilities, whereas the physical world operates only upon brute facts.

9. [SUM] Why does Wittgenstein say that this is not a relation in the ordinary sense? He is now dismantling the theory of knowledge defended at the time by Bertrand Russell. Russell thought that judgment (or belief) was an ordinary relation between a person and several objects. For example, in “John judges that Socrates is mortal”, Russell saw a direct multiple-term relation joining John, Socrates, and mortality. But for Wittgenstein, “ a judges that p ” is not a relation in the ordinary sense for two main reasons: (1) an ordinary relation occurs between objects, but p is not an object. If we try to replace the proposition p with a proper name, which designates an object, the proposition completely loses its sense. We can say “John judges that the Earth is round”, but we cannot say “John judges that Mary”. A judgment requires a proposition, something that expresses a possible fact, not a mere object or name. Propositions and names belong to entirely different logical categories. (2) Judgment is related to the duality of the proposition- true/false. When we judge that something is the case, our mind is positioning itself in relation to a diagram with two outcomes: either it is true or it is false. We are grasping one of the two possibilities at the expense of the other. That is why, in the case of judgment, the subject a is not merely connecting itself to a static thing. This is not an ordinary relation such as “John is to the right of Mary”, which either exists or does not exist. In judgment, the subject is relating itself to a proposition p that already comes, as it were, with two poles: true and false. It is a two-pole or double-direction structure, quite different from the proposition pure and simple. Judgment is a completely different logical animal.
10. [SUM] Wittgenstein was trying to create a purely geometrical and spatial notation to replace the logical algebra of Russell and Frege. To understand this tangle of letters and dashes, we must recover the concept of the poles a and b , where a is the pole of truth and b the pole of falsity, and combine it with the quantifiers of logic. What Wittgenstein is drawing in textual form are the logical connections that define the sense of a general proposition. Thus, in the first line of the notation, $a - (x) - a \phi x b - (\exists x) - b$, Wittgenstein is telling us how the true pole (a) and the false pole (b) of the whole proposition connect with the poles a and b of the internal function ϕx : the true pole (a) of the whole proposition is connected to the true pole (a) of ϕx through the operator (x) . That is, the whole sentence is true only if ϕx is true for every x . The false pole of the whole proposition is connected to the false pole (b) of ϕx through the operator $(\exists x)$. That is, it is enough for there to be at least one x for which it is false for the whole proposition to become false. In the second

line of the notation, $a - (\exists x) - a \phi x b - (x) - b$, Wittgenstein is telling us that the true pole (a) of the whole proposition is connected to the true pole (a) of ϕx through the operator $(\exists x)$. This means that the whole proposition will be true if we find at least one true case in the world. The false pole (b) of the whole proposition is connected to the false pole (b) of ϕx through the operator (x) . That is, the whole proposition will be strictly false only if all the x ’s examined are false. The crucial question here is why Wittgenstein went to the trouble of inventing all this strange notation full of dashes. In our view, he wanted to prove that logical operators (such as “all”, “there exists”, “not”, and “or”) are not things or objects in the world. In Russell’s logic, it seemed that “ $\exists$ ” was a concept that we added to the sentence. Wittgenstein denies this by showing that quantification is merely a retexturing of the connections between the true and the false. If we drew this on paper, as he in fact did in diagrams known as “ ab diagrams”, we would see lines linking the external poles to the internal poles. To understand the logic of a sentence, for the young Wittgenstein, was a matter of seeing this geometrical flow of possibilities. Everything was inside logic and its recursive operations, without any external or empirical foundation or support.

11. [SUM] What was Wittgenstein trying to achieve with his new geometrical notation, the notation of the a - b poles? The “old definitions” to which he refers are the logical definitions of Bertrand Russell and Gottlob Frege. In the traditional logic of these authors, in order to define a logical connective or a complex relation, it was necessary to use axioms and rules of inference. For example, to prove that the negation of negation returns to the starting point ($\sim \sim p \equiv p$), or to define the conditional “if p , then q ” in terms of negation and disjunction ($\sim p \vee q$), a whole apparatus of logical laws was required. When Wittgenstein introduces his method of the a and b poles, the rules of the game change completely for two fundamental reasons: (1) logical constants are eliminated, since, for Russell, symbols such as $\sim$ (not), $\vee$ (or), and $\supset$ (implies) were “logical constants” - almost like abstract objects that operated upon sentences. Wittgenstein destroys this idea by showing that these connectives are merely rules for mapping paths between the poles of truth (a) and falsity (b). The moment we draw the connections and the flow of logical possibilities, as we saw in the example of the quantifiers, we perceive that logic is purely combinatorial. (2) What used to be a “discovery” is now obvious from the notation itself. In a perfect notation based on poles, and later in the truth tables of the TLP, if we apply the operation of negation twice to p , the drawing of the lines simply throws us back to the original pole of p . No axiom or “law of double negation” is needed to prove this. The notation itself shows that the two things are identical. To say that the old definitions become tautological means that they reveal their true nature: they express no deep truth about thought or about the world; they are merely repetitions of structures already given in language itself. They say the obvious ($A = A$) on which all logic rests. For Wittgenstein, the logic of Russell and Frege was full of “formulae” that they believed to be profound philosophical truths. With the analysis of the a - b poles, Wittgenstein shows that all these formulae are merely pseudo-propositions that say nothing (tautologies); gears turning in the void, whose equivalence is demonstrated visually and mechanically by the very structure of language.
12. [SUM] The proposition “facts are symbolized by facts” marks the inauguration of one of the most famous theories in the entire history of the philosophy of language: the *picture theory of meaning*, which Wittgenstein would later consolidate in the TLP. The geometrical and spatial configuration of words within a proposition, which is itself a fact, generates the sense, which points to another fact. In the proposition “The inkwell is on the table”, what symbolizes is the fact that the word “inkwell” has been placed in a specific spatial relation, to the left of, or before, the expression “is on the table”. There is a structure, a living architecture in the proposition. The words are staging a relation. What relation? A relation between objects in the real world that also configures a fact. In order to represent this real fact, language cannot use only static objects; it must create a con-

figuration, a fact, within the symbol itself. Thus, in “The inkwell is on the table”, the word “inkwell” appears arranged in a certain way next to the word “table”, a linguistic fact that serves to project and represent the possibility of the real inkwell being on the real table, a fact of the world. For the young Wittgenstein, language is a dynamic mirror of reality. A proposition can describe a fact of the world only because it is itself, in its logical and grammatical structure, a fact.

13. [SUM] What kind of criticism does Wittgenstein make here of the theory of types? In this passage, the criticism of Bertrand Russell’s theory of types is subtle but devastating. It follows the maxim that “the shot backfires”: for Wittgenstein, Russell was absolutely right in his fundamental intuition, but badly mistaken in thinking that he needed to create a “theory” full of artificial rules to solve the problem. The theory of types was created to save logic from the paradox of the set of all sets that do not contain themselves. The paradox is that this set both contains and does not contain itself. To solve this problem, Russell proposed a rigid hierarchy of types, in which a proposition of a higher type cannot speak of itself, thereby avoiding the logical short circuit. We would thus have Type 0, of individual objects, for example “Socrates”; Type 1, of properties of objects, for example “Socrates is mortal”; Type 2, of properties of properties, and so on. When Wittgenstein says that “A proposition cannot occur in itself. This is the fundamental truth of the theory of types”, he is offering a critique of syntactic self-sufficiency. He argues that Russell treated logic as if it were a traffic system in need of guards and prohibition signs: “No self-referential propositions!” Wittgenstein says: no, the very nature of the symbol prevents this from happening. We do not need a theory to prohibit the impossible. For our author, a proposition p is a sign that expresses a sense, a possibility of the world; for p to occur within itself, something like $p(p)$, the outer p would have to be a function and the inner p would have to be the argument, the object. But the same sign cannot at once be the whole function and the piece that enters into it, because they have entirely different logical natures: one names, the other describes. The criticism, therefore, is that the theory of types is redundant and badly formulated. Russell thought he was creating a complex mathematical rule, but he was only stammering a truth already given in the deep grammar of language itself. If we try to write a proposition that contains itself, we do not generate a forbidden sentence; we generate only a tangle of letters with no sense at all. Language protects itself; it does not need a philosophical theory to police what can or cannot be said.
14. [SUM] A classic example that fits Wittgenstein’s definition perfectly would be the proposition “This is red”. If we think of the deep structure of this proposition according to the logic of the NL, it illustrates exactly the three points of the paragraph: (1) It says something indefinable about a thing: the sensation or pure property of color, such as “red”, is a general indefinable. We cannot define what the color red is by using logical concepts or by decomposing it into more basic linguistic subcomponents. We can point to it or show it, but in language it functions as an irreducible logical block, a simple predicate. The word “this”, or the name a , points to the thing, and “red” is what is said of it. Therefore, the proposition has the classic subject-predicate structure (φa) . (2) The proposition is recognized by containing only one name and one form: respectively, “this”, which locates the object in space and time, and the property “is red”, which is attributed to the object by the very form of the proposition. In other words, we do not need an instruction manual or an external theory to know what kind of proposition this is. Merely by looking at the architecture of the symbol, we recognize that it is a pure subject-predicate proposition. And here lies Wittgenstein’s checkmate against Russell’s theory of types: Russell thought we needed to create a law saying, “The predicate ‘red’ belongs to Type 1 and the object ‘this’ belongs to Type 0; therefore they may combine”. Wittgenstein says that this is obvious simply by looking at the symbol. The “type” or logical category of the proposition is already stamped on the face of the symbol itself. We recognize its logical nature by its geometry and by the way its words fit together, without any need for

a logical traffic guard to dictate the rules.

15. [SUM] This paragraph is the perfect complement and logical conclusion of the previous line of reasoning. If Wittgenstein explained that the type of a simple proposition, such as a proposition of the “subject-predicate” type, is recognized directly in its own symbol, here he extends this rule to the more complex propositions of logic: those that use variables and quantifiers, what in Russell’s time was called an “apparent variable”. When, for example, we want to say something general in logic, such as “All human beings are mortal”, we use variables: $(x) \cdot Hx \supset M$. For Wittgenstein, this means two things: (1) our universal proposition must mention a type of proposition; that is, the quantifier symbol (x) does not operate in a void. It must look toward a mold, a prototype of a proposition; it points to the form of the proposition being modified, for example the subject-predicate form φx . (2) Our general proposition must show which components are constant; the symbol must make clear what is fixed and what is changing. If I say that “There exists an x that is red”, the property “being red” is constant, the concept at issue, while x is the variable, the geometrical place that can be occupied by any object. The notation itself, visually, separates the fixed frame from the mutable content. Content constants, predicates and names, are therefore the fixed components of a sentence that have meaning in the real world, such as “red”, “Socrates”, and “mortal”. The NL take these to be the “indefinables” that give body to the proposition. They determine the type of the proposition. As we saw in the previous paragraph, if we already know what the constant components are, we immediately know the logical category of the proposition. We should note, however, that content constants, predicates and names, are *different from logical constants*. Symbols such as $\sim$ (not), $\vee$ (or), $\supset$ (implies), and (x) (all), are logical constants. Frege and Russell thought that these symbols were names of real and abstract logical objects or functions. But, for Wittgenstein, logical constants are merely indices of operation, that is, notational tools for connecting and rearranging the poles of truth (a) and falsity (b) of propositions. With this, our author anticipates what in the TLP he would call his “fundamental idea”: there are no logical constants.
16. [SUM] In the logic of the time, the notation $(\varphi) \cdot \varphi!x$ means: “For every simple property φ , x has that property”. Faced with a statement like this, Russell would be led to think that if the proposition speaks about all the properties of x , then it has itself just created a new complex property for x , the property of “having all properties”. Therefore, this proposition is trying to include itself in the group of things about which it is speaking. It was precisely to avoid this vicious circle that Russell created the theory of types. But Wittgenstein is saying here, against all Russell’s fears, that the very description of the symbol blocks the vicious circle. If we describe the kind of symbols expressed by $\varphi!$, and if, as argued, this is sufficient to determine the type, then automatically “ $(\varphi) \cdot \varphi!x$ ” cannot be covered by that description. The mold or model that defines what a simple property $\varphi!$ requires symbols of a certain irreducible type, as we saw in the example of “is red”. The whole proposition $(\varphi) \cdot \varphi!x$ is a complex proposition, containing an operator of totality (φ) and a variable. It is a completely different logical animal. It does not fit inside the mold of $\varphi!$. Therefore, when the proposition says “for all properties φ ”, it is referring strictly to those properties that fit the mold of $\varphi!$. Since the whole proposition itself has a completely different geometrical structure, it is automatically excluded from the batch. It cannot speak about itself, even if it wanted to.
17. [MS1] Let us recall what has already been mentioned earlier in the *Presentation*: the themes found in the *Summary* and in MS2 recur in MSS 1, 3, and 4. We accept Potter’s hypothesis (2009, pp. 265–271) that the *Summary* and MS2, because of their very similar writing style, should be transcriptions of what Wittgenstein explained in person to Russell about his ideas between October 8 and 9, 1913. Much of what is in MS2 also seems to be a reformulation of what is contained in MSS 1, 3, and 4. These manuscripts, in turn, would be English translations of the typescript produced in Birmingham, now lost. Thus, in this first paragraph, we have an articulation among

indefinables, names, forms, and propositions. Wittgenstein stitches these four concepts into a single, perfectly integrated mechanism. The correlation he establishes among them is structural and visual. How does it work? First, indefinables are the raw material of this architecture. For Wittgenstein, reality is composed of simple things, and language, consequently, must have elements that can no longer be divided or analyzed: these are the *indefinables*. They are the fundamental pieces of the game and are divided into two kinds: *particular indefinables* (the objects or things of the world), and *general indefinables*: properties (colors, shapes) and relations (such as “being to the right of” or “being greater than”). Second, names and forms are the molds of indefinables. Each kind of indefinable is projected into language through a specific symbolic category: the *particular indefinable* is represented by a name (“this”, “a”, “Socrates”, etc.). The name merely points to the simple object; the *general indefinable* is what gives rise to the form of the proposition (e.g., the subject-predicate form ϕx , or the relational form xRy). The form is the geometrical frame, the empty “mold” that determines how names may fit together. Third, the proposition is the resulting symbolic fact. When we take a form (an empty logical mold) and fill its gaps with names (which point to objects), we generate a *proposition*. The proposition is not a mere heap of words; it is a living configuration. The fact that the names are arranged within that specific form creates a three-dimensional mirror that projects a sense (a possibility of being true or false in the world). Because of this direct correlation, Wittgenstein states that the type of a proposition can be recognized simply by looking at its symbol. We do not need an external theory (such as Russell’s) to classify propositions; the very structure of names and forms within the proposition explains and protects itself. We should also observe that later, in the TLP, Wittgenstein abandons the concept of “indefinable”. Although the TLP solves exactly the same problems that Wittgenstein was groping toward in the NL, it undergoes a profound conceptual and terminological purification. The word “indefinable” (which he had inherited from Russell and Frege) is replaced by a much more precise and elegant terminology based on the idea of *simplicity*. For the Wittgenstein of the TLP, to say that something is “indefinable” still carries a residue of Russellian philosophy, as if the mind needed to “intuit” some abstract and mysterious object that cannot be explained. By replacing “indefinable” with “simple sign” (name), Wittgenstein shifts the weight back onto language and the method of analysis: if we analyze a complex proposition to the end, we will inevitably arrive at names that point to simple objects. If the sign is simple, it cannot be defined because it has no parts; it is the endpoint of linguistic analysis.

18. [MS1] What is the peculiarity emphasized in the idea that propositions are also facts, that is, in the example given that the inkwell is on the table? Basically, the fact that the real inkwell is on the real table can only be expressed by a corresponding linguistic fact, and not by a heap of names. This idea points to two fundamental points: (1) words are not “labels” stuck onto objects - the tendency of traditional logic was to think that, in order to describe the inkwell on the table, we would have to take the name “inkwell”, the name “table”, and the name of the relation “on”, and join them together. Wittgenstein shows that this would generate only a list of three things: inkwell, relation-on, table. A list of objects says nothing; it is merely a complex of names. For there to be sense, the word “inkwell” must actually be written in a certain spatial and logical position in relation to the word “table”. The symbol is not a thing; the symbol is the fact that the words are organized in that way on the page. (2) The spatial relation must be shown; the peculiarity of the inkwell example is that language stages reality. In order to symbolize a fact of the world (a physical object in a certain relation to another), language uses a physical fact of its own (a written sign in a certain spatial relation to another). It is the very geometrical structure of the proposition that says how the world is. Let us think of the physical arrangement described by the paragraph: the symbol expresses the physical object inkwell positioned on the physical object table: this is fact *a*; fact *b* is

the sense or world also expressed: the physical object Ludwig seated on the physical object chair. In this scenario, the inkwell on the table is not communicating something about itself. It has been transformed into a pictorial fact. The spatial relation between the inkwell and the table (“being on”) is being used to mirror and symbolize the spatial relation between Wittgenstein and the chair (“being seated on”). It may be useful to recall here that roughly thirty years later, Wittgenstein said twice in the PI (§§ 15 and 26) that yes, “naming is similar to attaching a label to a thing”. What changed between then and later? Answering this question correctly helps, in my view, to clarify how he used logic roughly between 1913 and 1929. Thus, if in 1913 the refusal of the name as a “label” represented an idealization of syntax and structure as self-sufficient operations, in the PI, by contrast, the metaphor of the “label” serves to show how we often use names as a proper prior preparation for use in concrete situations of our everyday lived life. For this reason, the most important theme in PI § 26, for example, is the question “But what is it a preparation for?” This shows that the self-sufficiency of logic as the regulative ordering of our practical activities loses primacy to the *praxiological situation itself* as the starting point of our practical activities. It is the latter that, in reality, delimits or circumscribes which logic, or which language-game, we shall use to deal with some problem, or to reach some decision at that moment. If in the NL the aim was to repair the language of logicians such as Russell and Frege by showing that a correct notation would reveal types and tautologies transparently, eliminating paradoxes once and for all, in the PI logic no longer serves to build a perfect system, but to perform linguistic therapy. When he says in §15 that “it is useful, when doing philosophy, to say to ourselves: naming something is rather like attaching a label”, he is using this image as a simplified model (a primitive language-game) to untie the conceptual knots we ourselves create when we try to philosophize in the abstract.

19. [MS1] Wittgenstein is combating the temptation to think that we can resolve or express a real fact of the world merely by changing the way we write or manipulate our symbols. Imagine two objects in the real world that share a common feature - for example, two apples that are red. Wittgenstein argues that if we tried to show that they have something in common by giving them the same name in two different textual ways (for example, writing one in italics, *apple*, and the other in bold, **apple**, or using the same name in two distinct grammatical contexts), we would be falling into an illusion. His argument is devastatingly simple: names are arbitrary. If the common property lay in the way I choose to draw or designate the name, it would be enough to change the name of one of them to “fruit-X” and the other to “fruit-Y” for the common feature to disappear from language. But in the real world, the two apples would continue to be red! When he says that “we are always tempted, in a difficulty, to take refuge in different ways of designation”, he is criticizing logicians such as Frege and Russell. When they encountered a conceptual problem in logic, they tended to invent a new notational distinction (such as creating assertion signs, changing the types of symbols, or creating different “ways of designating” the same object) in order to force language to express what it cannot express by decree. How was this same issue resolved in the TLP? In the TLP, this intuition from the NL is elevated to the status of one of its most famous axioms, formally resolved through the distinction between what language says and what language shows. Wittgenstein resolves the issue by decreeing that the internal properties and common features of objects cannot be said or expressed by names or propositions. They manifest themselves in the structure of language. In § 4.1211, we read: “Thus a proposition ‘*fa*’ shows that the object *a* occurs in its sense; two propositions ‘*fa*’ and ‘*ga*’ show that in them one is speaking of the same object”. This is complemented by § 5.53: “I express the identity of the object by the identity of the sign, and not by means of a sign of identity. I express the difference of objects by the difference of signs”. There is one name for each object: if two objects are different, they receive strictly different names (*a* and *b*); we never use “different ways of designating” the same object. And the common

feature shows itself: if two objects share a common feature (a property), this will be mirrored by the fact that their different names can fit into the same logical mold (the same predicate) - if I write Fa and Fb , the common feature between objects a and b is not being “said” by a special rule of designation; it shows itself visually by the fact that both names can combine with the same function F . If we tried to invent a symbol or a trick of designation to “explain” what a and b have in common, we would be trying to say what can only be shown, thereby producing nonsense. The language of the TLP is a crystalline mirror: what is common in the world is reflected as common in the geometry of propositions, without notational tricks.

20. [MS1] Wittgenstein points out an error that, for him, contaminated the foundations of the logic of his two great mentors. The central error he criticizes here is categorial reductionism: both Frege and Russell tried to treat a proposition as if it were a kind of name (that is, a label for a “thing”), failing to understand what logical sense is. For Frege, an entire proposition functioned essentially as a complex proper name whose meaning was a truth-value: either true or false. Thus the proposition “Snow is white” would be merely an elegant name for the abstract object “the true”. But if a proposition were only the name of an object (even if that object is truth), it would be static. A proper name has no direction; it does not express a possibility. A proper name merely points to a thing. As we saw before, a proposition has two poles (ab): it comes ready-made with the possibility of being true or false. If the proposition were the name of the true, the negation of that proposition would be logically incomprehensible (how would one negate an object?). For Wittgenstein, a proposition names nothing; it describes a state of affairs. Russell, on the other hand, held that “propositions correspond to complexes”, that they dealt with complex facts of the world. A true proposition corresponded to a real “complex” (a combination of objects in the world, such as the book physically being on the table). But the problem with Russell’s thesis arises when the proposition is false. If the book is not on the table, the corresponding “complex” simply does not exist in the real world. If the sense of a proposition depended on its corresponding to a complex, then every false proposition would lose its correspondent and therefore lose its meaning, becoming pure nonsense. But we understand false propositions perfectly well! Wittgenstein realizes that a proposition does not need a real complex in order to have sense; it only needs to present a logical possibility of combination.
21. [MS1] In a *reductio ad absurdum* argument, Wittgenstein is demonstrating here that general structures and logical relations cannot be treated as if they were objects or names. General propositions such as “ $(\exists x, \varphi). \varphi x$ ” (There exists an object x that has a property φ) or “ $(\exists x, y). xRy$ ” (There exist two objects x and y that stand in a relation R) mention no specific object of the world. They deal only with variables and forms. For this reason, Russell and other logicians were tempted to think: “Well, if these propositions have no visible complex parts (names of real objects), then they must be simple symbols”. From there, they thought that the first was merely the name of a form and the second the name of a relation. They treated logical structure itself as if it were an abstract “thing” that language was naming. Wittgenstein sees that if we treat these general formulas as if they were names, the whole of logic collapses the moment we try to use negation. Consider ordinary proper names such as “John” or “Lisbon”. Does it make logical sense to place the operator “not” before them? Can we say “Not-John” or “Not-Lisbon” expecting to obtain a proposition with a truth-value? Obviously not. “Not” ($\sim$) can only be placed before something that expresses a proposition; that is, something that has two poles, that proposes a possibility that can be true or false. If the expression “ $(\exists x, y). xRy$ ” were merely the name of a relation, placing the sign of negation before it would be as nonsensical as saying “not-chair” or “not-table”. With this argument, Wittgenstein proves two fundamental points for his philosophy: first, that general propositions are also complex; even if they do not mention specific objects (such as a or b), they possess an

internal structural and geometrical complexity. They express a sense, map possibilities, and are therefore legitimate propositions, not names. And second, that logic names nothing; logical forms and relations are not abstract “super-objects” floating in the universe waiting for logic to create names for them. The logical form of a relation is the way language articulates itself, not a thing to be named.

22. [MS1] In the NL the world is a set of facts, but truth and falsity are properties exclusively of our propositions. Reality, therefore, is constituted by states of affairs that may or may not be realized. For this reason, there are likewise no true and false facts, only positive and negative facts. A positive fact is the existence of a relation between objects in the real world. For example: the fact that the inkwell is physically on the table. Something is the case. A negative fact is the non-existence of a state of affairs. If the inkwell is not on the table, this is not simply “emptiness”; for Wittgenstein, the absence of that relation in real space is also a factual configuration of reality. The world is determined both by what is present in it and by what is absent from it. Both (positive and negative facts) belong to the brute physical world. They simply are. “Truth” and “falsity” enter the scene only when a human being attempts to describe the world through a proposition. The proposition already carries within itself the two exits: a and b . The great insight is that a fact cannot be “false”, because if it were false, it would be “nothing”, it would not be a fact. And a fact does not need to be called “true”, because it is already given reality itself. Truth and falsity are the result of the fit (or misfit) between the measuring tape of language and the fabric of reality. The world provides the facts (positive or negative); our language provides the judgments (true or false).
23. [MS2] As we have already said in the *Presentation*, MS2, because of the similarity of its themes to those found in the *Summary*, may have been the manuscript Wittgenstein took to Cambridge for his discussions with Russell in October 1913. In this first paragraph, then, we find a justification for the fact that we immediately understand propositions we have never heard before. For our author, it seems inevitable that we possess indefinable general symbols. In this context, that expression means the pure and abstract logical molds (the forms) that structure our thought. The “indefinable general symbols” are the point of articulation between the new and the old. When we hear an unprecedented proposition, such as “A red Ferrari electric car is parked on the summit of Mount Everest”, we are able to decipher it because it is constructed by combining two elements: (1) the names (particular indefinables), since we already know the meaning of isolated words such as “car”, “red”, and “summit”; and (2) the forms (indefinable general symbols), because we already master the hidden geometry of language, such as the pre-formatted mold that dictates how things relate in space and in logic (the subject-predicate interrelation, or the spatial relation xRy). These forms are called general symbols by Wittgenstein because they do not speak of any specific object; they serve for any object. And they are indefinable because we do not learn the logical structure of language through dictionary definitions; we simply master it intuitively when we learn to speak. In this way, Wittgenstein is led to think of the isomorphism between this pre-formatted structure and everyday linguistic practice in order to justify the understanding of meaning. Therefore, our capacity to understand the new depends on a geometrical syntax that is already installed in us. A new proposition is like a cake mold we already know very well (the indefinable general symbol), but filled with a flavor of batter we have never tasted before (the names). We may never have tasted that specific cake, but by looking at its shape, we immediately recognize what it is.
24. [MS2] When Wittgenstein says that what corresponds in reality to a composite proposition must not be greater than what corresponds to its atomic (simple) propositions, he is saying the following: logical connections (such as “AND”, “OR”, “NOT”, “IF... THEN”) add no new fact to the world. If we look at the world and see the fact p (“It is raining”), and look to the side and see q (“It is cold”), what corresponds in reality to these two atomic propositions are only these two brute

facts. If we join the two sentences and say: "It is raining AND it is cold" ($p \cdot q$) - what has changed in reality? Absolutely nothing. No third fact called "AND" has come into being, floating between the rain and the cold. Material reality would continue to contain exactly the same quantity of things: the rain and the cold. And if we say: "It is NOT raining" ($\sim p$), the corresponding reality has not increased either. We have not added an object called "Not" to the world; we have merely pointed to the absence of rain. Unlike what Russell supposed, logical connectives do not function as nouns that attach labels to new objects in the world. They are merely operations we perform in language in order to coordinate, combine, or filter the poles of truth (a) and falsity (b) of simple propositions. Logic is a conceptual frame that we apply over the facts; it does not alter the size, weight, or quantity of reality.

25. [MS2] This paragraph represents the apex of the emptying-out that Wittgenstein performs upon the nature of logic. It establishes logic's autonomy and absolute lack of commitment, separating it once and for all from any remnant of ontology or science. The idea that "Logic must not deal with [particular] things" was already a consensus among the logicians of the time (such as Russell and Frege). Logic does not deal with the material world. The proposition "If all men are mortal and Socrates is a man, then Socrates is mortal" is not a truth about Socrates or about human biology; it is a purely formal truth. Logic deals only with variables (x), which can be filled by anything in the universe. Therefore, particular objects are outside logic. However, the complement "...nor must it deal with relations and predicates" places Wittgenstein's revolutionary blow in the foreground. Logicians such as Russell thought that, although logic should not deal with particular things (such as "Socrates"), it should still deal with the pure forms of properties and relations (such as the abstract concept of "being red" or the relation itself of "being to the right of"). Russell thought that predicates and relations were universal "logical objects". Wittgenstein says: No. If logic deals with predicates and relations, it will still be trapped in the content of the world. But if we remove from logic objects, properties (predicates), and the connections of the world (relations), what remains? Only the pure combinatorial possibility of language remains. For Wittgenstein, logic is the empty skeleton of representation. It deals strictly with the rules of coordination between the poles of truth and falsity (ab) of propositions, without caring about what is being said. Logic cares only that, whatever we utter, it will assume the form of a proposition that can be true or false, and that can combine with others through connectives. Logic is not a "super-science" that studies abstract objects and relations in the heaven of concepts. Logic is purely syntactic, formal, and combinatorial. It does not deal with the components that fill the world; it is merely the mirror that shows how the pre-formatted molds of language fit together and cancel one another out (tautologies and contradictions). In short, we also do logic because we *want* to - but would this prominence of the will, which nullifies the reifications of form, not be a Schopenhauerian inscription within analytic philosophy?
26. [MS2] What are "real variables"? Russell divided variables into two categories: the first category are *apparent* (or *bound*) variables, which were variables captured by a quantifier (such as "All" or "There exists"). For example, in $(x).\phi x$ ("For every x , x is mortal"), x is an apparent variable because the operator (x) delimits and closes the sense of the proposition. The second category, *real* (or *free*) variables, were those that appear loose, without any quantifier binding them. For example, when I write only the isolated expression " ϕx " (" x is red" or " x is mortal"), we do not say here which x , whether "all" or "only some". For Russell, an expression with a real variable such as ϕx was still a proposition (he called it a "propositional function") and he thought that it could be true or false depending on the value we inserted in place of x . Wittgenstein radically rejects this idea. For him, an expression that contains a real (or free) variable is not a proposition because it has no sense. And it makes no sense precisely because it is not concrete, lacking any grounding in practice or

situational connection. A proposition must be a measuring tape applied to reality: it must come ready-made with two determined poles (the pole a of truth and the pole b of falsity). It must propose a specific state of affairs in the world, so that we can look at reality and judge whether it coincides or not with the formal drawing. The expression ϕx by itself affirms nothing, denies nothing, projects no possibility, and does not position itself in relation to the poles of truth. It is merely an incomplete mold, a form with a hole in the middle. It is a fragment of a proposition, but not a proposition in itself.

27. [MS2] The sentence "What corresponds in reality to a proposition depends on whether it is true or false" reserves, in my view, the philosophical space of complete autonomy for the logical structure of language, as well as for the exercise of the will in the *best Schopenhauerian sense* of the term, within the natural history of the human species. Why do I say this? It is enough to take into account that in the NL the proposition is always neutral: it provides for both the truth and the falsity of what it asserts; therefore, the discussion of the proposition's truth-value (whether it is a fact or not) becomes, for NL, a problem of the world, not of the structure of language. This is the answer of the NL to metaphysical realism, empiricism, metaphysical idealism, and solipsism in general. The radical neutrality of the proposition in the NL, the idea that language is only a two-pole geometrical map (ab), and that the world sovereignly decides, through the facts, which pole will be activated, is the point of support Wittgenstein uses to implode some of the great currents of traditional philosophy. Metaphysical realism, empiricism, idealism, and solipsism commit exactly the same basic error: they all try to use language to take a position before or beyond the facts, mixing the neutral structure of the mirror with what is being mirrored - operationally neutral regarding the tape measure against which we judge reality. So, his theory would deal with each of these currents in a very specific and demolishing way. (1) With *metaphysical realism*: reality is not a logical "guarantee". Metaphysical realism claims that the world exists independently of our mind and that there are "real structures" or "universals" (such as necessary properties and relations) that language names. Wittgenstein's verdict in this case would be: realism is right to note that the world decides the truth-value of propositions, but badly mistaken in thinking that logic can "prove" or "guarantee" the existence of those structures. Since the proposition is neutral, logic does not care which facts actually exist. If logic depended on the real existence of objects or complexes in order to have sense (as Russell thought), falsity would be impossible. Therefore, logic cannot be metaphysically realist; it is purely formal and combinatorial. It simply accepts the world. (2) With *empiricism*: sense precedes experience. Empiricism holds that all knowledge and all meaning come from sensory experience and from the ascertainment of the facts of the world. Wittgenstein's verdict in this case would be: this is where "neutrality" delivers its most subtle blow. Wittgenstein reverses empiricism: in order for us even to have an experience and verify whether the inkwell is on the table, we must understand the proposition beforehand. Sense (the two-pole configuration) must already be ready in language before the world decides whether it is true or false. Empiricism deals with the truth of propositions, but their sense is purely logical and aprioristic. We do not learn what logic is by experiencing the world; logic is what makes experience possible. (3) With *metaphysical idealism*: the world does not depend on thought in order to be a fact. Metaphysical idealism claims that reality is, in some way, dependent on the subject's mind or on the categories of thought (the world is representation). Wittgenstein's verdict in this case would be: idealism tries to invade the space of the world. If the proposition is neutral and provides for both its truth and its falsity, this means that language merely projects the space of possibilities. Thought does not have the power to "create" the positive or negative fact. The subject establishes the measuring tape (the proposition), but the length of the table belongs to the table, not to the measuring tape. Idealism confuses the form of language (which is ours) with the verdict of

the world (which is independent of us). (4) With *solipsism*: the “I” is not a fact of the world. Solipsism takes idealism to the extreme: “Only I exist; the world is my judgment”. Wittgenstein’s verdict in this case would be: in the NL, when he says that “assertion is merely psychological” and that “the proposition ‘*a* judges that *p*’ is not a relation in the ordinary sense”, he is cutting the legs from under solipsism. The “I” that judges, believes, or has psychological states is only an empirical subject studied by psychology; that is, it is only one more detail inside the world - it is not the limit of the world. The solipsist tries to say “The world is my representation”, but in order to say this, he must use a proposition. And since every proposition is neutral, the solipsist’s proposition would have to provide for its opposite (“The world is not my representation”). At the moment when the solipsist tries to transform his view into a necessary logical truth, he produces nonsense, for logic takes no side. How is all this resolved in the TLP? This posture of the NL prepares the ground for the famous resolution Wittgenstein will give in the TLP (§§ 5.6 to 5.641), where he makes realism and idealism meet and cancel each other out: the space of language (the logical molds that contain the *ab* poles) is strictly the limit of my world. For this reason, idealism and solipsism have a correct intuition: the boundaries of language are the boundaries of the world. However, this truth cannot be said by any proposition; it only shows itself. At the moment when the solipsist or the realist tries to write philosophical theses about the world, he tries to make neutral language say a metaphysical necessity. Language jams, and the result is silence or nonsense. With this, we can understand how the *rhinoceros problem* stands here: on 11/01/1911, Russell comments in a letter to Lady Ottoline Morrell about the young Wittgenstein: “My German was very argumentative and tiresome. He wouldn’t admit that it was certain that there was not a rhinoceros in the room (...)” (McGuinness, 1988, p. 89). From the point of view of logical neutrality, what does this mean? Could this position not have a certain kinship with a kind of Schopenhauerian anti-realism? The famous anecdote of the rhinoceros in the classroom at Cambridge (probably in Russell’s room at Trinity College) takes on a completely new meaning when illuminated by the radical neutrality of logic that Wittgenstein would formulate shortly afterward, in 1913. Far from being childish stubbornness or a crisis of Pyrrhonian skepticism (as Russell initially thought), the young Wittgenstein’s refusal to admit that “it was certain that there was not a rhinoceros in the room” sheds light on the heart of his break with Russell and reveals a deep structural kinship with anti-realism - although in a strictly logical guise, one that dialogues directly with Schopenhauer. The philosopher of the will to life opens his greatest book with the famous Cartesian maxim taken to the extreme: “The world is my representation”. For him, the things of the external world do not exist as independent “things in themselves”; they are objects structured by the *a priori* molds of our intellect (space, time, and causality). The phenomenal world is a construction of the subject’s cognitive mesh. If Wittgenstein had yielded to Russell and admitted that the absence of the rhinoceros was a “logical certainty”, he would have allowed empirical experience to dictate the rules of what can or cannot be thought. For neutral logic, the idealized rhinoceros in the sentence and the real flesh-and-blood rhinoceros have exactly the same status of possibility. Logic must keep the door open to the rhinoceros, even if the room is empty. When we draw the parallel between *The World as Will and Representation* and the NL, we see a “linguistification” of Schopenhauer: logical space in place of the space of representation; what Schopenhauer called “forms of representation”, Wittgenstein translates into logical forms (the indefinable general symbols). The world can present itself to us only if it fits into the molds of our language. Whether or not the rhinoceros is in the room is merely an accidental fluctuation of the “world as fact”. But the configuration that makes it possible to think the rhinoceros belongs entirely to the structure of the logical subject. Wittgenstein was not therefore telling Russell that an invisible or magical rhinoceros could be there - that would be vulgar mysticism. He was saying that Russell’s certainty was of the wrong kind.

Russell was trying to use physical and empirical certainty to close the logical space of possibilities. By refusing Russell’s verdict, Wittgenstein was defending the Schopenhauerian isolation of form: the world of brute facts may do whatever it wants, but it will never be allowed to dictate or limit the boundaries of what our language is capable of projecting as possible.

28. [MS2] Here Wittgenstein attacks the heart of Russell’s strategy, showing that one cannot use language (predicates or properties) to create logical divisions that ought to be purely structural. If we try to distinguish Type 0 from Type 1 by saying that the members of Type 0 are “simple” and those of Type 1 “are not simple”, we fall into a fatal logical trap. Why? In order to say that a property φ , such as “being simple”, is denied of the members of Type 1, the expression $\sim\varphi(x)$, where x is of Type 1, would have to be an expression that is either false or true, but meaningful. But if the expression has sense, this means that the logical category of x allows it either to receive or not to receive that property. In other words, we have placed the member of Type 1 under the same conceptual umbrella as Type 0. If we can apply the same predicate to both, even if negatively, we have not separated them into different types; we have only treated them as objects of the same type that possess different physical or factual characteristics. What the NL are defending here is the self-sufficiency of the symbol. We can never distinguish types by attributing or denying properties to them, for the same reason that we cannot place the sign “Not” before a proper name. Properties and predicates serve to speak about the facts of the world: whether the inkwell is or is not on the table, and never to speak about the categories of logic itself.
29. [MS2] If a logical system allows us to choose different paths for defining its fundamental building blocks, this is proof that we have not yet reached the true elementary bedrock of language. In Russell’s logic, if we wanted to define the elementary logical operations, there was a choice. We could, for example, take “NOT” ($\sim$) and “OR” ($\vee$) as primitives, or indefinables, and from them define “IF... THEN” ($p \supset q \equiv \sim p \vee q$); or else we could do the reverse: take “IF... THEN” and “NOT” as primitives and, from them, define “OR”. There was an alternative indefinability. What was a basic and “indefinable” block in one system became a derived and “definable” element in another, depending on the taste or convenience of the logician. Wittgenstein finds this malleability unacceptable. For him, logic is not a game of human conventions or a system of axioms that we invent and organize as we please. In the NL, logic is the crystalline and unique skeleton of language and the world. If “NOT” and “OR” were truly indefinable, that is, irreducible logical atoms, it would be a conceptual impossibility to try to define them through something else. This dissatisfaction in the NL was the driving force that led Wittgenstein, in the TLP, to create his famous Operation *N* (joint negation, the Sheffer operator / NAND / NOR). In his later work, he proved that there is a single fundamental operation from which all of logic can be generated. When we reach that deep level, “alternative indefinability” disappears.
30. [MS2] The temptation of Russell’s logic was to treat logical properties and relations as if they were predicates that we apply to propositions. For example, when we say that a proposition is “true”, “false”, or “plausible”, it seems that we are attributing a predicate to it in the same way that we attribute “red” to a physical object. Wittgenstein says that this is impossible “because of sense”. As we have seen, a proposition is born with two poles (*ab*): it is a neutral three-dimensional configuration that projects a possibility in logical space. The predicates and relations of the physical world, such as “being greater than” or “being blue”, serve strictly to connect objects to one another within the configuration. They operate inside the state of affairs. A whole proposition is not an object; it is a symbolic fact, a measuring tape. We cannot apply a predicate of reality to the very measuring tape that measures reality. To try to say that the proposition *p* “has the property of being true” is a category error, for truth is not a physical property of the sentence; it is only the geometrical fit between the poles of the proposition and the facts of the world. Thus, “Not”

and “Or” introduce nothing new. If these indefinables were real predicates or relations, every time we used them in a composite proposition we would be adding new content or new objects to the universe. For Wittgenstein, in fact, logic introduces nothing new into reality.

31. [MS2] A proposition without names is a purely general or formal proposition, constructed by means of variables and logical quantifiers. For example, $(\exists x, y). xRy$ or $(\exists x, \varphi). \varphi x$. These propositions mention no individual or specific object in the universe. They do not contain “names” in the strict sense. They are made only of models, variables, and logical operators. But why do they remain “complex”? Because these general propositions do not contain names of objects, they may be confused with “simple symbols”, as if they were the name of an abstract form. But Wittgenstein thought this was a mistake. They are complex because they possess articulated parts that perform a precise syntactic function: first, because they divide logical space, since they propose a situation that may be true or false. In order to have this double outcome, they must necessarily be complex. Second, because they possess an internal geometry: in the proposition $(\exists x, y). xRy$, for example, the symbol shows us a structure: there are two poles of identification, x and y , and a direction of relation, R , which goes from one to the other. There is a web, a network of logical relations taking place within it. For Wittgenstein, language never operates by means of simple and massive blocks. A proposition is never a disguised name. Even when language rises to the level of maximum and purely general abstraction, i.e., without citing a single object of the real world, it continues to act as a model or a complex logical architecture. What makes it a proposition is not the bricks, the names, that we place inside it, but rather the structure of geometrical relations that allows it to serve as a measuring tape for reality.
32. [MS3] In this paragraph we see that Wittgenstein seems to return to points that we have already seen discussed in this text. But the reader should consider that MS3 probably belongs to a manuscript distinct from MSS 2 and 4, and from the *Summary*. There is, therefore, a relative autonomy among the manuscripts and the *Summary*. What is being resumed here? First, the distinction between positive and negative facts; second, the precedence of sense over truth (a critique of empiricism); the critique of Frege, when he says that “being true” is not a predicate that we can place outside a proposition, since propositions are not names for “truth-values”. This passage, incidentally, eventually became one of the most famous passages of the TLP, in section § 4.063. A beautiful metaphor (the black spot on the white paper) illustrates what we have been discussing up to this point. But there are also new elements in this paragraph. In addition to the metaphor of the spot on the paper, we also have two elements that are new up to this point: (1) the diagnosis of “nonsense”; and (2) predicative self-sufficiency. As for (1), the nonsensical proposition is not the one that is false, but the one that fails to locate a point on the paper. It fails to establish a coordinate. Since it points nowhere, we cannot even say that it is false or true, because there is no “thing” there to receive that judgment. As for (2), if we already have the proposition p (“The inkwell is on the table”), we do not need to write “It is true that the inkwell is on the table” for it to function. The original proposition already contains within itself the articulation (the predicate “being on”) that makes it capable of being true or false.
33. [MS3] At the root of this paragraph, we might ask what philosophical conception of “reality” Wittgenstein is handling here (pretending, for the sake of argument, that Wittgenstein were professing some philosophical position). Do we have a *transcendental idealism*? Perhaps yes; but a strictly linguistic one. There is an undeniable kinship with Kant’s transcendental idealism and, above all, with Schopenhauer’s rereading of it. Space, time, and the categories are *a priori* molds of our cognitive apparatus. The real world (the phenomena) is forced to conform to these forms so that we can perceive it. Do we have *solipsism*? Perhaps yes; but a solipsism without a subject. Traditional solipsism says: “Only I exist; the world is my illusion”. Wittgenstein’s solipsism is radi-

cally different: it is a logical solipsism. When we admit that reality is determined by the negations of propositions, we are saying that the limits of language are the limits of the world. But of which language? Of the language that I can understand. However, the reader should notice the paradox: if the limits of language coincide perfectly with the limits of the world, the “I” (the subject that thinks) disappears. There is no “I” outside the world looking at it, nor an “I” inside the world operating as a thing. The subject shrinks until it becomes a geometrical point without extension - it becomes the very limit of the world, and not a part of it. At the moment when the solipsistic subject disappears, what remains? Only reality itself, coordinated with it. For this reason solipsism becomes pure realism. Finally, do we have a *constructive empiricism*? The answer, peremptorily, is *no*. An empirical-constructive view (such as the one the logical positivists would try to assemble years later, or such as psychological theories of knowledge do) suggests that we look at the raw data of the senses and, brick by brick, “construct” a model of reality. For Wittgenstein, we do not construct logical space through experience. Logical space is already entirely given at the moment when we babble the first meaningful sentence. We do not “construct” reality; language merely provides the pixel grid (the retina of the eyes), and the physical world simply turns those pixels on or off through facts. If we had to give this posture a philosophical name, the best one would be a logical-transcendental realism. But the reader should observe carefully that Wittgenstein’s entire attitude, from this period onward, was to escape philosophy. “Reality” here is only a determined limit. It is completely operative, indicating the final snapshot of logical space after the application of all the prefabricated molds of language, with special emphasis on the ontological weight of absences: the world is the totality of positive facts plus the closure imposed by negative facts. To know how reality is requires knowing exactly what it is *not*.

34. [MS3] Why does Wittgenstein say here that “this is not enough to justify the inference”? This passage from the NL strikes a mortal blow at the heart of Frege’s and Russell’s logical foundationalism. When Wittgenstein states that their laws of deduction “are not enough to justify the inference”, he is saying that logic does not need external “traffic laws” to give us permission to pass from one proposition to another. For Wittgenstein, logical inference is self-justifying, based entirely on the internal geometry of the propositions themselves. And these, from the standpoint of symbolism, are simply the way we act: we apply a tape measure to reality to measure it and regard it *as such*.
35. [MS3] This passage from the NL consolidates and gives a technical name to what we have been discussing concerning free (real) variables. Here Wittgenstein draws a surgical distinction between the instrument ready for use (the proposition) and the mold or template for its manufacture (the *propositional schema*). The fundamental difference between them lies in the presence or absence of sense and meaning. The propositional schema is like a blank check. It has all the necessary structure: the space for the name, the space for the amount, the line for the signature. But it is not a payment. One cannot buy anything with it because the fields are filled with empty variables. The *proposition*, by contrast, is the check filled out and signed. Whether because someone’s name has been placed there (a constant), or because a general rule valid for any bearer has been stamped on it (a quantifier), it now has exchange value. It has come to signify a real transaction. Wittgenstein’s target here is, once again, Bertrand Russell. By lowering propositional functions to the status of mere “schemata”, Wittgenstein is saying that Russell’s logic was full of ghosts: symbols that looked like propositions, but were only empty molds. For Wittgenstein, logic deals only with what is complete; either the symbol has sense and positions itself rigidly before the poles of truth, or it is merely a preparatory drawing on the logician’s drafting board.
36. [MS3] What is the criticism of Russell here? Wittgenstein indicates that Russell is trying to perform a kind of “ontological juggling”: he wants his complexes to have two contradictory natures

at the same time in order to save his logical system. In doing so, Russell ended up confusing what is a *property* (a fact of the world) with what is a *logical type* (a structure of language). Russell's *Theory of Types* rigidly prohibited entities of different categories from occupying the same place in syntax. A simple individual belongs to one type; a complex (a relation or proposition) belongs to a completely different type. If Russell says that a complex can be "treated as simple", he is breaking his own rule. For it to make sense to say of a simple object that it "has become a complex", the sentence "This simple is a complex" would have to be a legitimate and meaningful proposition. But, according to Russell's own logic, this is an absolute contradiction in sense. A simple and a complex are incompatible logical types. With the thesis that "a property cannot be a logical type", Wittgenstein delivered his checkmate. Russell treated "simplicity" and "complexity" as if they were physical properties of objects; as if they were "being blue" or "being heavy". He thought that an object was born simple and that, by combining with others, it acquired the "property" of being complex. Wittgenstein corrects his master: being simple or being complex is not a property that an object possesses; it is its very logical type. If something is a complex (an articulation of parts), it can never, under any circumstance or "perspective", be treated as a simple.

37. [MS3] If "OR" can be completely dissolved and transformed into "NOT", and "NOT" can be transformed into something else through these cross-definitions, this proves conclusively that neither of them possesses real substance or denotes a relation in the world. If they denoted real relations, they would be irreducible, just as "being taller than" or "being to the left of" are irreducible in language. Since OR and NOT accept this dance of notational substitutions, it is proved that they are only tools of writing, operational signs that we invent to manipulate the poles of propositions, and not components of reality.
38. [MS3] If we imagine a concrete atomic proposition, such as "Socrates is mortal" (ϕa), and remove from it the particular constituent a ("Socrates"), replacing it with a variable x , we clean out the subject, leaving only the mold ϕx ("x is mortal"). From this mold we can generate an entire class of propositions (which is what the formula $\hat{p} \{(\exists x). \phi(x) = p\}$ states: the set of all propositions saying that something is mortal ("Plato is mortal", "The dog is mortal", etc.). Wittgenstein notes that this class is still partly arbitrary. Why? Because it still depends on our dictionary convention about what the symbol ϕ means. We choose, by pure convention, that ϕ means "being mortal". If we changed the human convention to "being blue", the generated class would be completely different ("The sky is blue", "The sea is blue"). But if we replace with variables all the symbols whose meaning has been determined by convention, the mold becomes something purely abstract, such as "x has the property F" ($F(x)$). Now the class of propositions that fits here is no longer the set of mortal things, nor of blue things. It is the class of all possible subject-predicate propositions in the universe, regardless of what they are talking about. When we remove every human convention whatsoever (the names of objects and the meanings of properties), what remains is not "nothing". What remains is a pure geometrical structure. This final class of propositions no longer depends on whether human beings exist, whether death exists, or whether the sky is blue. It depends "only on the nature of the symbol". It shows how language, by its own internal syntax, divides space and creates relations. A logical type is not a label or tag that the logician pastes onto words. The logical type is the structural and invariable mold that remains in a proposition when we remove from it all semantics and the entire material world. It is the pure possibility of the articulation of language.
39. [MS3] If p and $\sim\sim p$ have the same sense, why do we write them in such different ways? In Russell's notation, the second proposition appears to be much more complex than the first. It contains two signs of negation, visually suggesting that we have "added" two operations or two new logical entities to the world. Worse still: Russell's system allows us to generate an infinite plurality of sym-

bols with the same sense. For Wittgenstein, this is evidence that Russell's symbolism is wrong. Pure logic should not accept redundancy, masking, or waste. If the structure of thought is one, ideal logical writing should be transparent and show this identity at once, rather than creating syntactic ghosts. For Wittgenstein, negation is not a "brick" or a property that we throw on top of a proposition. Negation functions exactly like a light switch: an operation that reverses poles. Russell's method deceives us because it makes us believe that logic advances and accumulates like mathematics, where $1 + 1 = 2$. Wittgenstein shows us that logic operates through balances of symmetry: if we negate twice, we do not construct a new complex proposition; we merely undo the previous movement.

40. [MS3] This passage from the NL offers a forceful critique of the illusion of complexity produced by Bertrand Russell's logical notation. Wittgenstein is saying that the traditional method of symbolization fails because it confuses the fat of writing (the signs we invent) with the real anatomy of thought (sense). When Wittgenstein says that names are not things, but classes, he is striking a blow against the naïve realism that tried to treat words as if they were concrete physical objects (ink marks on paper or sound waves in the air). Consider the example given: "A" is the same letter as "A". If we look at this proposition from the point of view of physics (of material "things"), it is strictly false. The first "A" printed on your screen occupies a different geometrical space from the second "A". If we were reading a printed book, the first "A" would be made of graphite-ink molecules different from those of the second. They are two physically distinct things. Yet, from the point of view of language, we read and understand both as the same letter. Why? Because what matters for logic is not the identity of the physical matter of the ink, but the syntactic function that this drawing performs within the system. The "name" or "letter" is the class that gathers all the infinitely many possible ink marks (whether handwritten, typed in italics, or in bold) that obey the same *rule of use*. But Wittgenstein also states that this is crucial because, if names were treated as individual physical objects (things), logic and symbolic language itself would instantly collapse. There are three major consequences here: (1) The possibility of repeatability of identity: for logic to function, we need to be able to repeat the same element in different parts of an equation. When we write a simple tautology such as $x = x$, this proposition has value only because the first x and the second x are recognized as the same symbol. If the first x were one physical thing and the second another physical thing, they would be two different objects, and the equation $x = x$ would be nothing but a physical lie. Logic requires the permanence of the symbol through the variation of matter. (2) Logic is different from psychology and physics: if the name is a class of signs sharing the same syntactic model, then the meaning of a word is not glued to the physical substance of the sign, nor to the mental image (psychological) we have in our heads when we read it. The symbol is purely formal. Logic operates at a level of abstraction that ignores whether we are thinking aloud, writing on paper, or running binary code on a computer; the functional class remains identical. (3) The clarification of notation: this reflection in the NL will flow into the famous section § 3.32 of the TLP: "The sign is the part of the symbol perceptible by the senses". Many of the errors of traditional philosophy and of Russell's logic occur because logicians allow themselves to be deceived by the fact that two completely different symbols in logic use the same physical sign in writing. Wittgenstein's classic example is the verb "to be": we use the same physical sign ("is") to denote the copula ("Socrates is mortal"), identity ("The ambassador is Maria"), and existence ("God is"). Physically, the word is the same; logically, they are three entirely distinct symbols (classes). Wittgenstein is warning us here not to confuse the material vehicle of language with its logical machinery.
41. [MS3] What Wittgenstein sets against Russell is the absolute autonomy of syntax. He argues that the possibility that a proposition makes sense (or is nonsense) must be guaranteed from within

language itself, and never by a psychological act or by external rules. In Russell's theory, judgment is like a shopping basket. Russell wanted to explain what happens in our mind when we make a false judgment (for example, if I judge that "The cat is in the box", but the cat is in the street). He did not want to admit that there existed a ghostly entity called a "false proposition" floating around. So he proposed that judgment is a multiple relation between the Subject (S) and the loose elements of the proposition. If I judge that "A loves B", my mind (S) operates as a knot that ties together three independent things: the object A, the relation *Love*, and the object B. Judgment functions like a shopping basket: the mind throws into it the subject, the object, and the verb. The verb ("to love") is treated merely as one more loose ingredient in the basket, stripped of its function of connecting things. The absurdity of "penholdding" lies in the fact that, if judgment is merely the mind tying loose elements together in a basket, what would prevent the mind from tying together elements that do not fit in any way? It could, for example, "penholder". To avoid this absurdity, Russell needed to appeal to his theory of types, an external rule coming from outside and saying: "The human mind is not allowed to do this because 'penholder' is not a verb". Wittgenstein's solution is that a correct theory of judgment must make this thought impossible in advance - and this is provided by the *a priori* mold of language itself. Judgment does not deal with loose pieces: when we judge, we do not throw objects and verbs loosely into a basket. We judge an entire proposition, which has already come from the factory with a rigid geometrical structure and with sense. Syntax takes care of itself: the sign "penholder" is the name of an object. It has a geometrical mold that allows it to fit only into name-slots. It simply does not possess the "hooks" required to occupy the place of a verb or of a relation.

42. [MS3] For Russell, connectives such as "OR" ($\vee$) and "IF... THEN" ($\supset$) are real Platonic logical relations that connect facts in the world. If this were true, when a scientist wrote an isolated molecular proposition such as $p \supset q$ ("If this liquid is heated, it expands"), he would be describing a solid, definitive, and autonomous fact about the logical structure of that liquid. But Wittgenstein notes that, in the sciences, an isolated proposition such as $p \supset q$ or $p \vee q$ is never regarded as a final scientific law. It is always provisional. If a physicist says, "If I press this button A, laser B fires", this factual implication is used only provisionally, so long as he does not yet know the general law behind the phenomenon. To say only $p \supset q$ functions almost as a confession of partial ignorance: we know that one thing follows from the other there, in that accidental case, but we have not yet captured the real mechanism of the world. When does an implication cease to be provisional and become a genuine scientific law? When it is placed under the scope of a universal quantifier (an apparent variable). The scientist is satisfied only when he transforms the isolated case into a general law of nature: $(x) . (\varphi x \supset \psi x)$. "OR" and "IF... THEN" are not an ontological cement that glues isolated facts together in the world. They are only rules of formal traffic that allow us to formulate generalizations. The connective acquires the dignity of a "law" only when it is bound to a sign of generality; the real function of logic in science is not to describe scattered "logical objects", but to provide the combinatorial mold that allows us to map the general laws of the universe.
43. [MS4] This passage shows the consolidation of Wittgenstein's entire logical-philosophical project in that period. In the sense that if everything we call the "world" were a screen made of pixels - or black spots arranged on a white sheet of paper (the atomic propositions, in this case) - it would be enough to turn each pixel on or off (to declare its truth or falsity) for the image of the world to be completely scanned. No ontological "remainder" would be left over. Later, in section § 4.26 of the TLP, Wittgenstein will say that "The specification of all true elementary propositions describes the world completely". However, the bracketed addition at the end of the paragraph - "[I doubt this.]" - is surgical. It already anticipates the cracks that would make this beautiful crystal palace collapse years later, with the problem of color exclusion. This issue demonstrated conclusively

that it was impossible to construct a system in which atomic propositions were independent of one another.

44. [MS4] The expression "my theory" appears in the *Summary* (p. A5) and also here, in MS4. It is interesting that, in 1913, Wittgenstein still conceived his philosophical activity through the expression "my theory", a usage that persisted at least until 1914, when he referred to his "Theorie der logischen Abbildung" (*Theory of Logical Picture*). Yet, by the time of the *Prototractatus*, a mere four years later, our young philosopher explicitly declares that "Philosophy is not a theory, but an activity", and also that "Logic is not a theory, but a mirror-image of the world" (MS 104, pp. 18, 66). These statements reappeared in the TLP in sections § 4.112 and § 6.13, respectively. Philosophy became an activity that, among its various characteristics, entails the critique of various theories, such as those of Russell and Frege. This was a development that emerged at the very dawn of his reflections, likely already fully compatible with what was being written in the NL.
45. [MS4] The formula " $\varphi a . \supset_{\varphi, a} . a = a$ " can be read as "For every property φ , and for every object a , if a has the property φ , then every a is identical with itself". But what Wittgenstein actually wants to say here is that there is no relation between the fact and its constituent; there is only a relation between fact and fact. The relation expressed by the sign of implication can occur only between two whole propositions, because both would possess, in this case, truth-poles. One cannot establish this relation of composition between, say, the brick and the wall, because one thing is part of the other; or between a pen and its blue color, where one is the object and the other a property. The formula above, therefore, is a "false theory of relations"; it expresses a deceptive similarity, a kind of disguise that makes two completely distinct things appear identical. This was exactly what Russell proposed with his theory of identity. In *Principia Mathematica*, we read that "If x and y are identical, any property of x is a property of y " (2019 [1910], p. 176). What Wittgenstein is doing with this formula is the diagnosis of an illusion. When he says that the similarity can be expressed by that formula, he is not validating the similarity as something real; he is merely showing the logical mechanism that deceives Russell. The formula is the perfect "disguise" that makes two completely distinct things appear identical on paper. It is, consequently, a *parody*, an *ironic imitation*.
46. [MS4] The reader must pay close attention to this passage because, in proposing the hypothetical premise that "a word creates a world", Wittgenstein is not defending any form of naïve linguistic idealism, the magical idea that words physically manufacture the things of the world, such as trees or tables, nor is he defending a preliminary form of deconstructionism, suggesting that "there is nothing outside the text". "A word creates a world" means only: "If we assume that a word has a sense and locates a valid possibility, then...". The moment we accept that a single word or proposition has sense, we are not adopting an isolated element. We are accepting the rules of the *entire game*. If a word is capable of making sense, it is because it is already immersed in a system where tautologies are true, where rules of inference operate, and where mathematics holds. Logic is a closed package: either we have everything, or we have nothing. The second half of the paragraph is what destroys any idealist interpretation: "...it could not create a world in which a proposition was true, without creating its constituents". For a proposition to be true (or even to have sense), its constituents (the simple objects of the world) must necessarily be there to serve as its stuffing. Language cannot invent a truth in a vacuum. What Wittgenstein is saying, therefore, is the opposite of idealism: language is hostage to the existence of objects. We cannot formulate a functioning proposition without the presupposition that logical space is already populated by the constituents that give it substance. Language does not "create" objects; language *presupposes* objects. The sense of language and the structure of the world are rigidly tied together by an *internal necessity*. This is a reflection on the limits of what can be thought logically, not on the magical creation of reality by words. It is a thought of strictly *transcendental* structure, in the most Kan-

tian sense of the term, although transposed onto the plane of language. The difference is that here the conditions of possibility of sense are located in the syntax of symbolism itself, in logical space - so to speak. However, none of this means - once again - that Wittgenstein is traditionally defending a "transcendental argument". We must understand that his posture, his philosophical attitude, from his very first written text onward, is entirely praxiological. His disposition is not philosophical, in the most academic and traditional sense of the word, but reduced strictly to the controversy over the philosophical foundation of linguistic practice. The whole inclination and behavior typical of what is philosophically traditional (such as attention to the relevant literature on the topic, detailed and rationally chained argument, the aim of persuasion), all this has always been disregarded by Wittgenstein's practice. What happens here, however, unlike the later stage of his thought, is that our author still conceived symbolism within the idealized limits of classical bivalent logic. This idealization led his construction to collapse after 1929, and his own conception of language also had to be praxiologized, so that *internal necessity* then became situational and multiple, depending on the normative context. The precept that if we accept the rule, we accept the entire game, the whole internal necessity of language, then becomes circumscribed to the different language-games in which we participate within a culture or, more precisely, within a form of life. Philosophical confusions no longer arise from the mistaken use of symbolism, but now from the mixing of rules from different language games.

47. [MS4] " $p \vee \sim p$ " is a tautology. Tautologies have no sense for Wittgenstein, because they declare nothing about the world. A tautology merely displays the structure of thought. But the proposition " $(p) \cdot p \vee \sim p$ ", in turn, applies to all possible propositions of logic. It is a universal law of logic called the "principle of excluded middle". It describes a structural property of symbolism itself; it declares something about the nature of language itself. The same applies to the *ab-functions*, because they merely demarcate the limits of the proposition, whose descriptive function applies only to what is true or false. The mistake of Russell and of the philosophical tradition was to think that, when we state a logical law, we are discovering a deep truth about the universe. Wittgenstein pulls us back down to earth: logic does not speak about things; it is only the empty mold into which things fit.
48. [MS4] Understanding exists only with the bipolar proposition, where there is knowledge of content. There is no knowledge with tautologies: an error Wittgenstein wants to combat, an error into which Russell frequently fell. Someone might argue: "Well, I know that the sentence 'The inkwell is on the table OR the inkwell is not on the table' is an absolute and necessary truth. Therefore, I understand the sentence 'The inkwell is on the table'". Wittgenstein says: not at all. Knowing that a tautology ($p \vee \sim p$) is true requires us to understand absolutely nothing about the real world. We may not know what a "inkwell" is, we may not know what a "table" is, and we may not know what the relation "R" means. Even in the most complete ignorance about the terms, we will still know, with 100% logical certainty, that "either aRb or not aRb is the case". The tautology is an armored truth, but it is armored precisely because it does not touch reality. It functions operationally within symbolism.
49. [MS4] For Russell, " xRy " was the form of a "dual complex" (1992, p. 98). This "form" was also one of the multiple constituents of the act of judging, together with the "subject" and the "objects". Judgment, for this author, involves a "multiple relation". When Othello judges that Desdemona loves Cassio, Othello's mind tied together Desdemona, Cassio, Love, and the abstract form " xRy ". The function of the form would be to serve as a template for showing who was the subject and who was the object in the relation. For Wittgenstein, however, form was not a thing to be placed among other things. A logical form is not an object of the world, not an entity; it is only the possibility of the structure of a proposition. Moreover, we cannot say that there is a relation between

forms and things in symbolism either. By bringing together form, an element of logical syntax, with things, objects of the world, Russell's theory of judgment produced a kind of hybrid monster. It transformed the mirror of language into one of the things reflected in it.

50. [MS4] In this section of the NL, Wittgenstein directs his artillery against the way Russell tried to explain the so-called *propositional attitudes*. The central criticism here is that Russell could not solve the psychology of judgment and belief because he did not understand what the *a priori* mold of a proposition is, its architecture. Russell tried to treat the believed sentence (p) as if it were an object or a name, forgetting that a proposition is an essentially bipolar structure. When A believes in p , what is at stake is not meaning (the object, the fact), but sense. p is not there to function as the name of a massive fact. p is there as a proposal of sense, a situation that may or may not occur. The act of believing is not a direct line between the mind and an object. To believe is to adopt a geopolitical orientation within logical space. When A believes in p , A's mind is positioning itself in relation to these two poles simultaneously. It points to pole a as the real pole and rejects pole b . Russell's theory was incapable of expressing this because, for Russell, negation ("not- p ") was an element added afterward. Wittgenstein shows that negation is already contained in the very bipolar nature of the original proposition. If we understand p , we already automatically understand what the falsity of p means.
51. [MS4] Of these six statements on the nature of philosophy that constitute what in this typescript was classified as "Preliminary" the TLP adopts, in a certain way, all the suggestions except the one that discusses metaphysics: "Philosophy consists of logic and metaphysics". Section § 6.53 of the TLP explicitly states that "...whenever someone wished to say something metaphysical, to show him that he had failed to give meaning to certain signs in his propositions". There is also, of course, a certain tension with the statement in the NL that philosophy is "purely descriptive". The TLP avoids saying that philosophy is "descriptive". Perhaps because, in the system of the book, it is the natural sciences that describe reality and facts. Philosophy comes to be defined as an activity of clarification and elucidation (TLP § 4.112). However, the idea that philosophy is "descriptive" returns with full force in the post-Tractarian period, as does the idea that "distrust of grammar is the first requirement for doing philosophy" - if sections §§ 371 and 373 of the *Philosophical Investigations* state, respectively, that "Essence is expressed by grammar" and that "Grammar tells what kind of object anything is. (Theology as grammar)," and if the role of philosophy is *conceptual investigation*, then distrust of grammar is crucial for constructing a picture that envisions the application of these rules within their specific contexts.
52. [MS4] Wittgenstein returns with another irony directed at Russell through the expression "theory of manufactured relations". The old error of logic, which was to squeeze all propositions of human thought into the *Subject-Predicate* form, is replaced by a new error: thinking that every proposition in the universe must express a relation. In Russell's logic, relations are not only simple entities that we find ready-made in nature (such as "being to the left of"). Russell created a conceptual machine capable of mass-producing - or manufacturing - artificial relations of any level of complexity. Russell did this by defining relations through propositional functions with more than one variable, as when saying, for example, that " $xRy \equiv \phi(x, y)$ ". These manufactured relations are merely scaffolding for writing (notational devices), not the real anatomy of thought. Russell was confusing the mechanical capacity of his symbolism to generate relational molds with the ontological structure of the world. But logic is not a tangle of cables. Treating every proposition as a relation makes it appear that logic is a substance that glues objects to one another through relational links. What defines a proposition for Wittgenstein is not the fact that it has a predicate or a relation inside it. What defines it is its essential bipolarity (the a - p - b diagram). A proposition is a model that can be true or false.

53. [MS4] This page is a handwritten addendum (the handwriting is not Wittgenstein's) to the present typescript. The Roman-numeral headings name the topics that run along the margins of all the pages of the typescript. It is an attempt to give a supposedly rational ordering to material that probably seemed confusing to Bertrand Russell.
54. [MS4] This page is also another handwritten addendum (the handwriting is by the same person as on the previous page) to the present typescript. This brace diagram represents the bipolarity of the proposition $\sim p \vee \sim q$. It is the somewhat more confusing precursor of today's "truth tables". The TLP still preserves this brace structure. (Cf. note 3 above).
55. [MS4] This third handwritten page is also attached to TS201a2. But I cannot make out what it represents or means.

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